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PROPERTIES OF CONTINUOUS ANALOG ESTIMATORS FOR A DISCRETE RELIABILITY GROWTH MODEL

by

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Properties of Continuous Analog Estimators

for a Discrete Reliability Growth Model

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Reader Aids -

Purpose: Widen state of the art

Special math needed for explanations: Probability and statistics, asymptotic theory

Special math needed to use results: Probability and statistics

Results useful to: Reliability engineers, reliability and statistical theoreticians

Abstract - A discrete reliability growth model (appropriate for success-failure data) whose derivation parallels that of a popular nonhomogeneous Poisson process model (appropriate for continuous failure time data) is considered. Following Finkelstein [1], continuous analog estimators are defined for use with the discrete model when there is a constant prespecified number of test trials between system configuration changes. The large-sample properties of these estimators, including s-consistency and s-normality, are established. Large-sample standard error formulas and confidence interval procedures are also developed.

1. INTRODUCTION

A reliability growth methodology (including a model specifying the pattern of reliability growth and accompanying statistical procedures) is an evaluation tool for estimating the current reliability of a system that has been repeatedly tested, redesigned, and retested. During the course of a reliability growth testing program, each redesign defines a new configuration of the system. As the testing progresses, more failure modes are observed and more fixes are implemented, thereby increasing the reliability. At the completion of testing, an estimate of the reliability can be obtained using only the data for the last configuration of the system. However, the sample size for the last configuration may be small (particularly if testing is expensive) and the resulting estimate of reliability may be very imprecise. The motivation for using reliability growth methodology is to increase the precision of the reliability estimate by utilizing all of the available test data -- from the last configuration tested as well as from all previous configurations.

A nonhomogenous Poisson process (NHPP) reliability growth model (also known as Duane model) is often used for planning and monitoring growth for systems whose reliability is characterized by mean-(operating) time-between-failures (MTBF) [2-5]. In this continuous-time-model, the time-varying MTBF is proportional to a fixed power of time. In other words, the model assumes a linear relationship on a log-log scale of the s-expected cumulative number of failures versus the cumulative time on test. The continuous Duane model has been extensively developed [2-22].

This report concentrates on a particular discrete reliability growth model suitable for reliability data on the performance of one-shot systems (e.g., a missile or a torpedo). These data typically consist of discrete (success or failure) observations. Finkelstein [1] and Crow [23] derive this model by assuming that, on a log-log scale, the s-expected number of cumulative failures is linearly related to the cumulative test trial number.

This assumption is identical to the assumption underlying the derivation of the Duane model except that "time-to-test" is replaced by "test trial number". The resulting model therefore can be viewed as a discrete analog to the continuous Duane model.

Finkelstein [1] considered estimation of the two parameters β and λ in this discrete reliability growth model under the premise that one item is tested for each system configuration. Among the estimators examined in his simulation study, the continuous analog estimators (CAE's) are particularly appealing because of their simplicity and an intuitive motivation drawn from the structure of the maximum likelihood estimators (MLE's) under the continuous NHPP model. The other competing estimators were either artificial or found to be numerically unstable, and none emerged as superior to the CAE's.

Aside from an intuitive motivation, little is known about the properties of the CAE's. Even the s-consistency property has only been conjectured from a simulation study, and no sound procedures for constructing large-sample confidence intervals are available. The object of this article is to fill this void. In addition to proving s-consistency, we also establish the asymptotic s-normality of the CAE's β^* and λ^* and thus set the basis for computing large-sample confidence intervals. One important finding of our study concerns the individual rates of convergence to normality for β^* and λ^*. They are different, parameter dependent, and are quite unlike the rates for the corresponding estimators under the continuous model. These results should serve as a warning that inference procedures developed for the continuous model may be inappropriate for the discrete model even though the estimators may be structurally similar.

Section 2 describes the discrete reliability growth model. Section 3 presents the CAE's and reviews what is known about their small-sample properties. Asymptotic properties and approximate confidence interval procedures are provided in Section 4 for the case of a constant prespecified number of test trials between system configuration changes (detailed proofs are given in the Appendix). The examples in Section 5 illustrate the confidence interval procedures.

Notation

MTBF	mean-(operating) time-between-failure
NHPP	nonhomogeneous Poisson process
CAE	continuous analog estimator
MLE	maximum likelihood estimator
λ, β	parameters of the discrete reliability growth model
R_i	system reliability for configuration i
n_i	number of trials for configuration i
T_i	$\sum_{j=1}^{i} n_j$, cumulative number of trials through configuration i
N	number of different system configruations tested
m	constant number of trials per configuration
λ^*, β^*	CAE's of λ, β
Y	total number of observed failures
f(j)	configuration at which the j-th failure occurred

Other standard notation is given in "Information for Readers & Authors" at the rear of each issue.

2. DESCRIPTION OF THE MODEL

For the discrete reliability growth model examined in this article, the reliability of the system under test increases with the configuration number i according to

$$R_i = 1 - \lambda[i^{\beta} - (i-1)^{\beta}], \quad i = 1,2,\cdots,N, \tag{1}$$

where $\lambda > 0$ and $\beta > 0$ are unknown parameters, and a new configuration of the system occurs after every m trials when system design changes are implemented. Although Finkelstein [1] only treated $m = 1$, multiple trials are more common in practice, and the properties of the estimators can be handled analytically just as easily provided that the number of replications is equal for all configurations. This situation could arise, for example, when the system producer delivers equal-sized batches for sequential phases of testing. The model (1) can also be obtained from Crow [23] whose more general formulation accommodates unequal numbers of test trials per configuration. Beginning with his model

$$R_i = 1 - \lambda' n_i^{-1}[(T_i)^{\beta'} - (T_{i-1})^{\beta'}], \quad i = 1,2,\cdots,N, \tag{2}$$

and identifying $n_i = m$ and $T_i = im$, $i = 1,2,\cdots,N$, the correspondences $\beta = \beta'$ and $\lambda = \lambda' m^{\beta'-1}$ follow directly.

From (1) it is apparent that the reliability is a decreasing function of both λ and β. The parameter λ is the system unreliability for the initial configuration. The parameter $(1-\beta)$ is often referred to as the "growth parameter" in the literature. Reliability growth occurs iff $\beta < 1$. Equation (1) also shows that as the number of system configuration changes grows large, the reliability increases towards the limiting value of 1. For large N, the approximation

$$R_N \approx 1 - \lambda\beta N^{\beta-1} \tag{3}$$

can be used. The error ε_n in this approximation is $o(N^{-1})$, i.e., $N\varepsilon_n \to 0$.

3. CONTINUOUS ANALOG ESTIMATORS

Generalizing the estimators originally proposed by Finkelstein [1], the CAE's for the discrete reliability growth model (1) are

$$\beta^* = Y\left[\sum_{j=1}^{Y} \ln (N/f(j))\right]^{-1},$$

$$\lambda^* = Y/(mN^{\beta^*}). \tag{4}$$

The reliability R_N at the N-th configuration is estimated by direct substitution into (1). Thus

$$R_N^* = 1 - \lambda^*[N^{\beta^*} - (N-1)^{\beta^*}], \tag{5}$$

or using the approximation (3),

$$R_N^* \approx 1 - \lambda^* \beta^* N^{\beta^*-1}. \tag{6}$$

Although the CAE's (4) are structurally identical to the MLEs for the corresponding NHPP model (with "configuration number" playing the role of "time-on-test"), they are not the MLEs for the discrete reliability growth model (1). The MLE's for (1) have no closed-form representations, and their determination requires iterative computational procedures [1,23]. In the simulation study [1], Finkelstein reports that all attempts to obtain MLE's were unsuccessful.

The simulation study [1] also investigated the properties of the CAE's for the special case m = 1. It was observed that the CAE's, in general, overestimate β and underestimate the system reliability. In an example with the true $\beta = 0.800$, the average of the β^* values decreased gradually with increasing number of test trials to 0.864 after 300 trials. Despite the extremely slow approach, Finkelstein asserted the s-consistency of the CAE's.

The asymptotic properties stated in Section 4 and proved in the Appendix confirm Finkelstein's conjecture. The Appendix also establishes the limiting s-normality of β^* and λ^*. Based on these results, large-sample confidence interval procedures are developed in Section 4.

To date, the only confidence interval procedures that have been associated with the CAE's are purely ad hoc in nature and not based on any distributional theory, either exact or asymptotic. For instance, an example in MIL-HDBK-189 [2] calculates a lower confidence bound for system reliability using the methodology developed by Crow [13] for the analogous continuous reliability growth model. Without presentation of any supporting rationale, [2] states that this approach provides a good approximation when each n_i, $i = 1,2,\cdots,N$, is large and the system reliability (presumably the final reliability) is high. The examples in Section 5 of this report illustrate that the choice between methodologies can give rise to potentially critical differences between computed lower confidence bounds for the final system reliability. In particular, the bound based on the large-sample results given in Section 4 differs from that based on the ad hoc adaptation of Crow's procedure for the NHPP model.

4. ASYMPTOTIC PROPERTIES AND APPROXIMATE CONFIDENCE INTERVALS

The asymptotic behavior of the CAE's is described in Theorems 1 and 2 (see the Appendix for the proofs). Theorem 1 establishes the s-consistency of the CAE's for the case of a constant prespecified number of trials per system configuration, thereby confirming the conjecture of Finkelstein [1]. In the same setting, Theorem 2 demonstrates that the asymptotic distributions of β^* and λ^* are s-normal. As the empirical evidence in [1] suggests, however, the rates of convergence of the CAE's are slow.

Theorem 1: For the discrete reliability growth model (1), the CAE's (4) are s-consistent.

Theorem 2: For the discrete reliability growth model (1), the normalized forms of the CAE's (4), defined as

$$Z_{1N} \equiv N^{\beta/2}(m\lambda\beta^{-2})^{1/2}(\beta^*-\beta),$$
$$Z_{2N} \equiv N^{\beta/2}(\ln N)^{-1}(m\lambda^{-1}\beta^{-2})^{1/2}(\lambda^*-\lambda), \tag{7}$$

are each asymptotically standard s-normal.

Theorem 2 indicates that the rates of convergence to s-normality for the two CAE's β^* and λ^* differ by a factor of $\ln N$. Moreover, both rates of convergence depend on the unknown true value of β. If $\beta = 1$, corresponding to no growth and i.i.d. observations, the rate of convergence for β^* is the usual $\sqrt{N}$. The rates of convergence for the CAE's behave quite differently than the rates for the corresponding estimators under the continuous time model (see [6,8,10,13]). Consequently, the inference procedures developed for the NHPP model are not appropriate for use with the discrete reliability growth model (1).

Confidence intervals for β and λ can be reasonably based on the approximate s-normal distributions of the standardized r.v.'s Z_{1N} and Z_{2N} defined in (7). Beginning with β, we consider the coefficient $g_N \equiv (m\lambda N^{\beta})^{1/2}$ of $(\beta^*-\beta)/\beta$ which involves the unknown parameters β and λ. To determine if asymptotic s-normality still holds when g_N is replaced by $g_N^* \equiv (m\lambda^* N^{\beta^*})^{1/2} = \sqrt{Y}$, write

$$\ln(g_N^*/g_N) = \frac{1}{2}(\beta^*-\beta)\ln N + \frac{1}{2}(\ln\lambda^* - \ln\lambda)$$

and observe that the r.h.s. converges in probability to 0 as a result of Theorem 1 and Theorem 2. Thus $g_N^*(\beta^*-\beta)/\beta \xrightarrow{d} N(0,1)$. As for λ, a similar argument shows that the coefficient $h_N \equiv (mN^{\beta})^{1/2}/(\beta \ln N)$ in Z_{2N} can be replaced by $h_N^* \equiv (mN^{\beta^*})^{1/2}/(\beta^* \ln N)$ with the result $h_N^*(\lambda^*-\lambda)/\lambda^{1/2} \xrightarrow{d}$

Therefore, large-sample 100(1-α)% confidence intervals for β and λ can be constructed as

$$\beta^*[1 + z_{\alpha/2}/\sqrt{Y}]^{-1} \leq \beta \leq \beta^*[1 - z_{\alpha/2}/\sqrt{Y}]^{-1},$$

$$(\lambda^*+d_N)-d_N(1+2\lambda^*/d_N)^{1/2} \leq \lambda \leq (\lambda^*+d_N)+d_N(1+2\lambda^*/d_N)^{1/2}, \quad (10)$$

where $d_N = (z_{\alpha/2}\beta^* \ln N)^2/(2m N^{\beta^*})$ and $z_{\alpha/2}$ is defined by $\text{gauf}(z_{\alpha/2}) = 1 - \frac{\alpha}{2}$.

These approximate confidence intervals can be employed to conduct large-sample statistical tests of null hypotheses of the form $\beta = \beta_0$ or $\lambda = \lambda_0$.

Finally, in the setting of a reliability demonstration test, it is desirable to specify a lower confidence bound for R_N, the system reliability at the completion of testing. In the Appendix, we show that a large-sample 100(1-α)% lower confidence bound for R_N is given by

$$R_N > 1 - (1-R_N^*)\exp[z_\alpha\sqrt{2/Y}]. \quad (11)$$

5. EXAMPLES

Two examples are presented in this section. Each assumes a total of 200 trials and 45 observed failures occurring at trial numbers 4, 6, 8, 9, 10, 12, 18, 20, 22, 25, 28, 30, 31, 33, 38, 42, 45, 46, 51, 53, 56, 59, 62, 66, 70, 75, 77, 82, 88, 99, 108, 109, 115, 119, 126, 134, 142, 148, 158, 161, 165, 172, 179, 183, 198.

Example 1: $m = 10$, $N = 20$

Using (4) - (6), the point estimates $\beta^* = 0.819$, $\lambda^* = 0.387$, and $R_{20}^* = 0.815$ are obtained. The results (10) - (11) provide the following 90% confidence limits: $0.658 < \beta < 1.085$, $0.214 < \lambda < 0.700$, and $R_{20} > 0.758$.

Example 2: $m = 1$, $N = 200$

Proceeding as described above, the point estimates are $\beta^* = 0.751$, $\lambda^* = 0.842$, and $R_{200}^* = 0.831$. The associated confidence limits are $0.603 < \beta < 0.945$, $0.328 < \lambda < 2.158$, and $R_{200} > 0.779$.

Discussion

In Example 1, the number of different system configurations is relatively small, the confidence interval for β is wide, and the interval contains $\beta = 1$ (corresponding to no growth). If the data had been analyzed using standard i.i.d. binomial techniques, the reliability estimate would have been 0.775 with an approximate 90% lower confidence bound of 0.737. Both of these values are less than their counterparts provided by the reliability growth methodology.

The combination of the two examples demonstrates the importance of correctly accounting for the test design when there are replications of each system configuration. If the given sequence of failure data had been observed with $m = 10$ and analyzed assuming $m = 1$, the estimate of final system reliability and the corresponding lower confidence bound would have erroneously been inflated. Differences of the magnitude indicated in the examples could be critical in the context of a reliability demonstration test in which producer compliance with prescribed reliability thresholds is under investigation.

Identical comments apply to the situation in which $m = 10$ and the data are analyzed according to the procedures developed by Crow [13] for the analogous continuous reliability growth model (see the discussion in Section 3). These procedures, which implicitly assume $m = 1$, would provide a reliability estimate of 0.831 and an associated lower confidence bound of 0.777.

APPENDIX

(Proofs for Asymptotic Results)

Additional Notation

We first relate the CAE's (4) to linear functions of some basic independent r.v.'s. Let X_i denote the failure count at the ith configuration, $i = 1,2,\cdots,N$, and let $f(j)$ identify the configuration at which the j-th failure occurs, $j = 1,2,\cdots,Y$. Then we have $Y = \sum_{i=1}^{N} X_i$ and $\sum_{j=1}^{Y} \ln f(j) = \sum_{i=1}^{N} X_i \ln i$. Define

$$V = \sum_{i=1}^{N} X_i \ln i,$$

$$T_{1N} = Y(\ln N) - V, \quad T_{2N} = Y,$$

$$U_{1N} = N^{-\beta/2}(T_{1N} - m\lambda\beta^{-1}N^{\beta}), \quad U_{2N} = N^{-\beta/2}(T_{2N} - m\lambda N^{\beta}). \tag{A.1}$$

We initially pursue the asymptotic properties of U_{1N} and U_{2N}, and then establish that the CAE's satisfy the asymptotic behavior described in Theorems 1 and 2.

<u>Lemma</u>: As $N \to \infty$, the limiting joint distribution of (U_{1n}, U_{2N}) is bivariate s-normal with mean 0 and covariance matrix

$$\Sigma = m\lambda \begin{pmatrix} 2\beta^{-2} & \beta^{-1} \\ \beta^{-1} & 1 \end{pmatrix}. \tag{A.2}$$

<u>Proof</u>: Let $q_i = 1 - p_i = \lambda[i^{\beta} - (i-1)^{\beta}]$ denote the unreliability for configuration i, $i = 1,2,\cdots,N$. From (A.1) we have $T_{1N} = -\sum_{i=1}^{N} X_i \ln(i/N)$ and $T_{2N} = \sum_{i=1}^{N} X_i$ where the X_i's are independent, $E(X_i) = mq_i$ and $\mathrm{Var}(X_i) = mp_iq_i$. Note that

$$\sum_{i=1}^{N} q_i = \lambda N^{\beta},$$

$$\sum_{i=1}^{N} q_i \ln(i/N) \sim -\lambda N^{\beta}\beta^{-1},$$

$$\sum_{i=1}^{N} q_i[\ln(i/N)]^2 \sim 2\lambda N^{\beta}\beta^{-2}, \tag{A.3}$$

Where the symbol ~ means that the ratio of the two sides tends to 1 as $N \to \infty$. The last two results in (A.3) follow from (3) and the integral approximation of a Riemann sum. Using these we have

$$\nu_{1N} \equiv E(T_{1N}) \sim m\lambda N^{\beta}\beta^{-1}, \quad \nu_{2N} \equiv E(T_{2N}) = m\lambda N^{\beta},$$
$$\sigma_{1N}^2 \equiv Var(T_{1N}) \sim 2m\lambda N^{\beta}\beta^{-2}, \quad \sigma_{2N}^2 \equiv Var(T_{2N}) \sim m\lambda N^{\beta}. \qquad (A.4)$$

Consider an arbitrary linear function $T_N = d_1T_{1N} + d_2T_{2N}$, $(d_1,d_2) \neq (0,0)$. We then have $T_N = \sum_{i=1}^{N} X_i[-d_1 \ln(i/N)+d_2]$ and

$$\nu_N \equiv E(T_N) \sim m\lambda(d_1\beta^{-1}+d_2)N^{\beta},$$
$$\sigma_N^2 \equiv Var(T_N) \sim m\lambda K^2N^{\beta},$$

where the constant $K^2 = [(d_1\beta^{-1}+d_2)^2+d_1^2\beta^{-2}]$. Next define

$$W_{iN} = N^{-\beta/2}[X_i-mq_i][d_2-d_1\ln(i/N)],$$
$$W_N = N^{-\beta/2}(T_N-\nu_N) = \sum_{i=1}^{N} W_{iN}.$$

Since $E(W_{iN}) = 0$, $Var(W_{1N}) \sim m\lambda K^2$, and

$N^{-\beta/2}|d_2-d_1 \ln(i/N)| < N^{-\beta/2}(|d_2| + |d_1| \ln N)$,

it follows that, given any $\varepsilon > 0$, there exists an $N_0(\varepsilon)$ such that $|W_{iN}| < \varepsilon(m\lambda)^{1/2}K$ with probability 1 for all $N > N_0(\varepsilon)$ and uniformly in i. Therefore, the Lindeberg-Feller central limit theorem applies. Thus $N^{-\beta/2}(T_{1N}-\nu_{1N},T_{2N}-\nu_{2N})$ is asymptotically bivariate s-normal with mean 0 and covariance matrix Σ given in (A.2). The variance terms were already derived in (A.4), and the covariance term follows in a similar manner. Finally, it can be verified that $N^{-\beta/2}(\nu_{1N}-m\lambda N^{\beta}\beta^{-1}) \to 0$, so the proof is concluded.

<u>Proofs of Theorem 1 and 2</u>

As Theorem 1 is a direct consequence of Theorem 2, it suffices to address the latter theorem. From (4) and (A.1) we have the relation

$$N^{\beta/2}(\beta^*-\beta) = (U_{2N}-\beta U_{1N})(T_{1N}/N^{\beta})^{-1}.$$

The Lemma entails that $(U_{2N}-\beta U_{1N})$ is asymptotically s-normal with mean 0 and variance $m\lambda$. Also since $T_{1N}/N^{\beta} = m\lambda\beta^{-1} + N^{-\beta/2}U_{1N}$ which converges in probability to $m\lambda\beta^{-1}$, we have $N^{\beta/2}(\beta^*-\beta) \xrightarrow{d} N(0,\beta^2(m\lambda)^{-1})$. Turning next to λ^*, (4) and (A.1) allow us to write

$$\begin{aligned} \ln \lambda^* &= \ln(T_{2N}/m) - (T_{2N}/T_{1N}) \ln N \\ &= \ln \lambda + \beta \ln N + \ln(1+c_N U_{2N}) - \beta(1+c_N U_{2N})(1+\beta c_N U_{1N})^{-1} \end{aligned} \qquad (A.5)$$

where $c_N = (m\lambda N^{\beta/2})^{-1}$. Noting that U_{1N} and U_{2N} are bounded in probability and $c_N \to 0$, (A.5) yields

$$N^{\beta/2}(mN)^{-1}(\ln \lambda^* - \ln \lambda) = \beta(m\lambda)^{-1}(\beta U_{1N} - U_{2N}) + o_p(1), \qquad (A.6)$$

and the r.h.s. has the limiting distribution $N(0,\beta^2(m\lambda)^{-1})$ as a consequence of the Lemma. Finally, use of the delta-method (see [24, pp. 385-6]) establishes that $N^{\beta/2}(\ln N)^{-1}(\lambda^*-\lambda) \xrightarrow{d} N(0,\beta^2\lambda m^{-1})$, which concludes the proof.

Derivation of Lower Confidence Bound for R_N

Beginning with the approximation (3), we initially estimate $\rho_N \equiv \lambda\beta N^{\beta-1}$ by $\rho_N^* = \lambda^*\beta^* N^{\beta^*-1}$. Following the same lines that developed (A.5) and (A.6) we obtain the representations

$$\ln(\rho_N^*/\rho_N) = 2\ln(1+c_N U_{2N}) - \ln(1+\beta c_N U_{1N}),$$

$$N^{\beta/2}\ln(\rho_N^*/\rho_N) = (m\lambda)^{-1}(2U_{2N} - \beta U_{1N}) + o_p(1).$$

An application of the Lemma then yields $N^{\beta/2}\ln(\rho_N^*/\rho_N) \xrightarrow{d} N(0,2(m\lambda)^{-1})$. Since $2(m\lambda^*)^{-1}$ is a s-consistent estimator of the limiting variance, we conclude that $b^* \ln(\rho_N^*/\rho_N) \xrightarrow{d} N(0,1)$ where $b^* = (\frac{1}{2} m\lambda^* N^{\beta^*})^{1/2} = (Y/2)^{1/2}$. Finally, to see that this leads to $b^* \ln[(1-R_N^*)/(1-R_N)] \xrightarrow{d} N(0,1)$ and (11), observe that both $N^{\beta^*/2}\ln[\rho_N/(1-R_N)]$ and $N^{\beta^*/2}\ln[\rho_N^*/(1-R_N^*)]$ converge in probability to 0 as a result of the s-consistency of the CAE's and the error bound reported for the approximation (3).

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Abstract - A discrete reliability growth model (appropriate for success-failure data) whose derivation parallels that of a popular nonhomogeneous Poisson process model (appropriate for continuous failure time data) is considered. Following Finkelstein [1], continuous analog estimators are defined for use with the discrete model when there is a constant prespecified number of test trials between system configuration changes. The large-sample properties of these estimators, including s-consistency and s-normality, are established. Large-sample standard error formulas and confidence interval procedures are also developed.