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by

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ABSTRACT

Locally most powerful similar (LMPS) rank tests are derived for the two-sample problem when the samples are type I censored on the right. Special parametric families are then considered to arrive at the censored-sample modifications of the Wilcoxon-Mann-Whitney (WMW) test, the Savage test and the median test from the point of view of LMPS as an exact optimality criterion. Asymptotic power function of the LMPS test is derived under a sequence of contiguous alternatives, and the censored sample modifications of the WMW test, due to Halperin (1960) and Gehan (1965), are compared to the LMPS test in terms of asymptotic efficiency. It is found that although Gehan's test is not exactly equivalent to the LMPS test, it is in fact asymptotically optimal, whereas Halperin's test is not even asymptotically optimal for the logistic location model.

Key words and Phrases: Two-sample rank test; Type I censoring; Locally most powerful similar; Asymptotic relative efficiency

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1. INTRODUCTION

This article considers the use of a finite-sample optimality criterion to construct rank tests for the two-sample problem when the data are type I censored. In the absence of censoring, the locally most powerful (LMP) criterion has played a major role in a large variety of nonparametric testing problems (cf. Hájek and Šidák, 1967). LMP rank tests under type II censoring has also been extensively treated in the literature (e.g. Johnson and Mehrotra (1972), Bhattacharyya and Mehrotra (1983)). With arbitrary censorship or random censorship schemes, this exact optimality criterion has proved elusive due to the complexity of the censored-rank likelihood. Consequently, recourse is often taken to asymptotic optimality, intuitive adaptations of the full-sample optimal rank tests to censored data, or use of some special likelihoods depending on the pattern of censoring. See, for instance, Halperin (1960), Gehan (1965), Peto and Peto (1972), Kalbfleisch and Prentice (1973), and Crowley (1974) for some of the relevant literature. Our object here is to show that when the samples are type I censored, an exact optimality criterion, namely, the property of locally most powerful similar (LMPS), can be employed to construct rank tests for the two-sample problem. We focus on a censoring scheme, called equal type I censoring, under which both samples are right-censored at the same fixed point T. The method can be readily extended to handle unequal or even multiple type I censoring.

Let $X_1,\cdots,X_{n_1}$ and $Y_1,\cdots,Y_{n_2}$ be independent random samples from absolutely continuous distribution functions F and G respectively, and let the combined sample order statistics be denoted by $W_1 < W_2 < \cdots < W_n$, where $n = n_1+n_2$. Under the equal type I censoring scheme, the observable data consist of $(\underset{\sim}{Z}_c,\underset{\sim}{W}_c)$ where $\underset{\sim}{W}_c = (W_1,\cdots,W_R)$, $\underset{\sim}{Z}_c = (Z_1,\cdots,Z_R)$ is the censored rank-order vector, $Z_i = 1$ (0) if W_i is an X (Y), and R denotes the total number of X_i's and Y_j's that are $\leq T$. Further, set $R_1 = \sum_{i=1}^{R} Z_i$ and $R_2 = \sum_{i=1}^{R} (1-Z_i)$ so $R_1+R_2 = R$.

Central to the formulation of a distribution-free test of the null hypothesis H_0: $F = G$ under the present censoring scheme, is the probability distribution of $\underset{\sim}{Z}_c$. First, the design of the equal type I censoring entails that the joint density of $(\underset{\sim}{Z}_c, \underset{\sim}{W}_c)$ is composed of the product of two multinomial probabilities -- the probability that r_1 (r_2) observations of the first (second) sample occur in the intervals (w_i, w_i+dw_i) corresponding to which $z_i = 1$ (0), and n_1-r_1 (n_2-r_2) observations of the first (second) sample occur in the interval (T,∞). Upon integrating the joint density with respect to $\underset{\sim}{w}_c$, we obtain the probability of $\underset{\sim}{z}_c$ as

$$P_{F,G}(\underset{\sim}{z}_c) = \frac{n_1!\,n_2!}{(n_1-r_1)!(n_2-r_2)!} \int \cdots \int_{\underset{\sim}{A}_r} \prod_{i=1}^{r} f^{z_i}(w_i) g^{1-z_i}(w_i) d\underset{\sim}{w}_c$$

$$\times [\bar{F}(T)]^{n_1-r_1} [\bar{G}(T)]^{n_2-r_2} \tag{1.1}$$

where f and g are respectively the pdf's of F and G, $\bar{F} = 1-F$, $\bar{G} = 1-G$ and $\underset{\sim}{A}_r = \{-\infty < w_1 < w_2 < \cdots < w_r \leq T\}$. Under H_0, the probability (1.1) reduces to

$$P_{H_0}(\underset{\sim}{z}_c) = \frac{\binom{n_1}{r_1}\binom{n_2}{r_2}}{\binom{r}{r_1}} F^r(T)[\bar{F}(T)]^{n-r}.$$

Note that, unlike the cases of no censoring or combined-sample type II censoring, here $\underset{\sim}{Z}_c$ is not distribution-free under H_0. However, R is a complete sufficient statistic for the nuisance parameter $F(T)$, and this fact suggests that a distribution-free test based on $\underset{\sim}{Z}_c$ must be conditional upon R. Along this line, Halperin (1960) proposed a heuristic modification of the

Wilcoxon-Mann-Whitney (WMW) test. His test statistic is of the form

$$W_H = \frac{1}{n^2}[W(R_1,R_2)+R_1(n_2-R_2)] \tag{1.2}$$

where $W(R_1,R_2)$ counts the number of times an X precedes an Y among the uncensored elements of the two samples, and the term $R_1(n_2-R_2)$ accounts for the total number of times an uncensored X precedes a censored Y. It is well known that in the absence of censoring, the WMW test is LMP for the logistic location alternatives. A natural question is then -- Does Halperin's intuitive modification retain the local power optimality under type I censoring, and if not, what form of the modification would achieve this? This question constitutes the major motivation of the present study.

Since R is a complete statistic, any similar test based on $\underset{\sim}{Z}_c$ must be conditional given R. Therefore, for rank tests based on type I censored data, we are essentially setting the optimality criterion that among similar level α rank tests, the power function is to be locally maximized. Such a test will be called a locally most powerful similar (LMPS) rank test.

A general form of the LMPS rank test is developed in Section 2. Special models are then considered in order to derive optimal modifications of the WMW test, the Savage test and the median test from the point of the LMPS criterion. It is found that the modified WMW test, due to Halperin, is not optimal under the logistic location model. Section 3 establishes asymptotic distribution and power of the LMPS test. Unlike the random censorship model which requires sophisticated machinery for the treatment of the asymptotics, we show that for the present case a fairly elementary treatment is possible by invoking a theorem of Sethuraman (1961) concerning the joint and conditional limiting distributions. In Section 4, we compare Halperin's test and another modification of the WMW test due to Gehan (1965), specialized to type I censoring, with our LMPS generalization of the WMW test in terms of their asymptotic efficiency. We find that Gehan's test is asymptotically, although not exactly, optimal, whereas Halperin's test is not even asymptotically optimal for the logistic location alternatives. Some extensions of the results of Section 2 are discussed in Section 5.

2. DERIVATION OF THE LMPS RANK TEST

In this section, we first derive the form of the LMPS rank test in a general setting and then consider its applications to some important models.

As with the derivation of the LMP rank tests, we consider a real-parameter family of alternatives: $F = F_{\theta_0}$, $G = F_\theta$, $\theta > \theta_0$, and for simplicity of notation, we take $\theta_0 = 0$. The parameter value θ_0 will often be suppressed in notation. For instance, $F(x)$ will stand for $F_0(x)$ and $E(\cdot)$ will denote the expectation under H_0: $\theta = 0$. We will use an upper dot for the first derivative with respect to θ.

Denote the conditional likelihood of $\underset{\sim}{z}_c$ given $R=r$ by $P_\theta(\underset{\sim}{z}_c|r)$ and let $S(\underset{\sim}{z}_c,r) = n^{-1}[\partial \ell og\ P_\theta(\underset{\sim}{z}_c|r)/\partial\theta]\Big|_{\theta=0}$. Then conditionally, given $R=r$, a level α test of the form $\chi(\underset{\sim}{z}_c,r) = 1,\ \gamma(r)$ or 0, if $S(\underset{\sim}{z}_c,r) \geq, =, < d_\alpha(r)$, with $\gamma(r)$ and $d_\alpha(r)$ determined by $E[\chi(\underset{\sim}{Z}_c,R)|r] = \alpha$, maximizes the conditional power, uniformly in a neighborhood B of 0, among all level α similar rank tests of H_0: $G = F$ against H_1: $G = F_\theta$, $\theta > 0$. (cf. Hájek and Šidák, 1967, Theorem II.4.8). To see that the conditional test $\chi(\underset{\sim}{z}_c,r)$ thus formulated also maximizes the unconditional local power, let χ' denote any other similar rank test of level α. Because R is complete, χ' must also have conditional level α given R and since χ has the maximum conditional local power, we have $E_\theta(\chi'|r) \leq E_\theta(\chi|r)$ for all $\theta \in B$ and all r, $0 < r \leq n$.

This in turn entails that $E_\theta[E_\theta(\chi'|R)]$, the unconditional power of χ', does not exceed $E_\theta[E_\theta(\chi|R)]$, the unconditional power of χ. Therefore, $\chi(\underset{\sim}{z}_c,r)$ is the LMPS rank test.

To obtain $P_\theta(\underset{\sim}{z}_c|r)$, we substitute $F_\theta(x)$ for $G(x)$ in (1.1) and divide by

$$P_\theta(R=r) = \sum_{s=a}^{b} \binom{n_1}{s}\binom{n_2}{r-s} F^s(T)[\bar{F}(T)]^{n_1-s} F_\theta^{\,r-s}(T)[\bar{F}_\theta(T)]^{n_2-r+s}$$

where $a = \max\{0, r-n_2\}$ and $b = \min\{n_1, r\}$. This gives

$$\begin{aligned} P_\theta(\underset{\sim}{z}_c|r) = {} & \{\binom{n_1}{r_1}\binom{n_2}{r_2}[F(T)/\bar{F}(T)]^{r_1}[\bar{F}_\theta(T)/F_\theta(T)]^{r_1} \\ & \times r_1!r_2! \int\cdots\int_{\underset{\sim}{A}_r} \prod_{i=1}^{r} [f(w_i)/F(T)]^{z_i}[f_\theta(w_i)/F_\theta(T)]^{1-z_i} d\underset{\sim}{w}_c\} \\ & \times \{ \sum_{s=a}^{b} \binom{n_1}{s}\binom{n_2}{r-s}[F(T)/\bar{F}(T)]^s[\bar{F}_\theta(T)/F_\theta(T)]^s\}^{-1} \ . \end{aligned} \qquad (2.1)$$

To arrive at a compact form of the test statistic $S(\underset{\sim}{z}_c,r)$, we assume that the family of densities $f_\theta(x)/F_\theta(T)$, $x \leq T$ satisfies the regularity conditions A1 formulated in Hájek and Šidák (1967, p 70). Let

$$\bar{R} = R/n \ , \quad \bar{R}_1 = R_1/n_1 \ , \quad \bar{R}_2 = R_2/n_2 \ ,$$

$$p = F(T) \ , \quad q = 1-p \ , \quad \lambda_n = n_1/n \ .$$

Also, for any given r, $1 \leq r \leq n$, we define the scores $a_r(i,f)$, $i = 1,2,\cdots, r$, corresponding to the truncated density $f(x)/p$, $x \leq T$ as

$$a_r(i,f) = E[\dot{f}(F^{-1}(pU_i))/f(F^{-1}(pU_i))] - \dot{F}(T)/p \qquad (2.2)$$

where $U_1 < U_2 < \cdots < U_r$ are the order statistics of a sample of size r from the uniform (0,1) distribution. Then, using (2.1), an evaluation of $\dot{P}_\theta(\underset{\sim}{z}_c|r)/P_\theta(\underset{\sim}{z}_c|r)$ at $\theta=0$, under the stated regularity conditions, leads to

$$S(\underset{\sim}{Z}_c,R) = \lambda_n(1-\lambda_n)(\bar{R}_1-\bar{R}_2)[-\dot{F}(T)/(pq)] + n^{-1}\sum_{i=1}^{R}(1-Z_i)a_R(i,f). \qquad (2.3)$$

The details are straightforward and hence omitted. Henceforth, for simplicity of notation, we will write S for $S(\underset{\sim}{z}_c, r)$ and χ for $\chi(\underset{\sim}{z}_c, r)$.

In the following we obtain some explicit results by specializing θ to either a location or a scale parameter. To apply the above general result to the location model $G(x) = F(x-\theta)$, we assume that $f(x)$ is absolutely continuous and that $\int_{-\infty}^{\infty} |f'(x)|dx < \infty$ where $f'(x) = df(x)/dx$. For this model, $\dot{F}(T) = -f(T)$ and $\dot{f}(x) = -f'(x)$. Using these into (2.2) and (2.3), we obtain the test statistic

$$S^* = \lambda_n(1-\lambda_n)(\overline{R}_1-\overline{R}_2)[f(T)/(pq)] + n^{-1}\sum_{i=1}^{R}(1-Z_i)a_R^*(i,f) \qquad (2.4)$$

with the scores $a_r^*(i,f) = E[\phi(U_i,f)]$ where

$$\phi(u,f) = -\frac{f'}{f}(F^{-1}(pu)) + \frac{f(T)}{p}, \quad u \in (0,1). \qquad (2.5)$$

As for the scale alternatives $G(x) = F(x\exp(-\theta))$, we assume that $f(x)$ is absolutely continuous and $\int_{-\infty}^{\infty} |xf'(x)|dx < \infty$. Here $\dot{F}(T) = -TF(T)$ and $\dot{f}(x)/f(x) = -1-xf'(x)/f(x)$. Corresponding to (2.4), we have

$$S_1^* = \lambda_n(1-\lambda_n)(\overline{R}_1-\overline{R}_2)[Tf(T)/(pq)] + n^{-1}\sum_{i=1}^{R}(1-Z_i)a_{1R}^*(i,f) \qquad (2.6)$$

where $a_{1r}^*(i,f) = E[\phi_1(u,f)]$ with

$$\phi_1(u,f) = -1-F^{-1}(pu)\frac{f'}{f}(F^{-1}(pu)) + \frac{Tf(T)}{p}, \quad u \in (0,1). \qquad (2.7)$$

Guided by these results it would be natural to formulate modifications of the WMW test, the median test, and the Savage test to equal type I censored data by considering the logistic and double-exponential location models and the exponential scale model, respectively.

Logistic-Location

For the logistic family $F_\theta(x) = (1+\exp[-(x-\theta)])^{-1}$, $\theta > 0$, we have $p = [1+\exp(-T)]^{-1}$ and $f(T) = \exp(-T)[1+\exp(-T)]^{-2}$ so $f(T)/p = 1-p$ and

$f(T)/(pq) = 1$. Also, using the property $-f'(F^{-1}(u))/f(F^{-1}(u)) = 2u-1$, $0 < u < 1$, the scores $a_r^*(i,f)$ reduce to

$$pE(2U_i-1) = p[\frac{2i}{(r+1)} - 1]$$

and hence the statistic (2.4) becomes

$$S_L \equiv \lambda_n(1-\lambda_n)(\bar{R}_1-\bar{R}_2) + \frac{1}{(1+e^{-T})}[\frac{2}{n(R+1)} \sum_{i=1}^{R} (1-Z_i)i - \frac{R_2}{n}]. \quad (2.8)$$

Since $\sum_{i=1}^{R} (1-Z_i)i$ is the WMW statistic based on the uncensored observations, S_L can be viewed as a modification of the WMW statistic under the present censoring scheme. Note that S_L is not equivalent to W_H of (1.2), the modification proposed by Halperin.

Double Exponential-Location

Taking $f_\theta(x) = ½ \exp(-|x-\theta|)$ and applying the approximate scores $a_r^*(i,f) = \phi(E(U_i),f)$, we would arrive at a modification of the median test. To obtain its form, we first use the property $-f'(F^{-1}(u))/f(F^{-1}(u)) = \text{sign}(u-½)$, where $\text{sign}(a) = 1(-1)$ if $a \geq 0$ (< 0), into $\phi(u,f)$ of (2.5). Substituting the resulting approximate scores into (2.4), we arrive at the following modified median test statistic:

$$S_D \equiv \lambda_n(1-\lambda_n)(\bar{R}_1-\bar{R}_2)[\frac{f(T)}{pq}] + \frac{1}{n}\sum_{i=1}^{R}(1-Z_i)[\ \text{sign}(\frac{pi}{R+1} - \frac{1}{2}) + \frac{f(T)}{p}\ ]$$

where $f(T) = ½ \exp(-T)$ and

$$p = \begin{cases} ½\ e^{-T} & , \ T < 0 \\ 1-½\ e^{-T} & , \ T > 0 . \end{cases}$$

Exponential-Scale

For the exponential scale alternatives $f_\theta(x) = \exp(-\theta)\exp[-x/\exp(\theta)]$, $\theta > 0$, we have $-f'(x)/f(x) = 1$. Thus, in view of (2.7), the scores $a_{1r}^*(i,f)$ are determined by $E[F^{-1}(pU_i)]$, which are readily obtained from the work of

Saleh et al. (1975). Using their results, together with $f(T) = \exp(-T)$, $p = 1-\exp(-T)$ and $Tf(T)/(pq) = T/p$, in (2.6) and simplifying, we arrive at the following modification of the Savage statistic:

$$S_E \equiv \frac{1}{n}[R_1(T+1)-R-\frac{RT}{p}(p-\frac{n_2}{n})] + \frac{1}{n}\sum_{i=1}^{R}(1-Z_i)\{R\binom{R-1}{i-1}\sum_{j=1}^{R}(-1)^{j-i}\binom{R-i}{j-i}(\frac{1}{j})$$

$$\times [T - \frac{1}{p^j}(T - \sum_{\nu=1}^{j}\frac{p^\nu}{\nu})]\} .$$

3. ASYMPTOTIC POWER

As is typical with censored sample rank tests, application of the LMPS test gets computationally involved with larger sample sizes so a large sample approximation is of interest. In this section, we derive the asymptotic distribution of the test statistic $S(\underset{\sim}{Z}_c, R)$ under a contiguous sequence of alternatives. The result is then used to establish the asymptotic power of the LMPS test. Unless specified otherwise, all limits are taken as $n \to \infty$.

For generality, we consider statistics of the form

$$S_n = c\lambda_n(1-\lambda_n)(\overline{R}_{1n}-\overline{R}_{2n}) + n^{-1}\sum_{i=1}^{R}(1-Z_i)a_{R_n}(i), \tag{3.1}$$

where c is a constant, $a_{R_n}(i)$ are some arbitrary scores, and n is used in the subscript to mark the sequence. The LMPS test is then a special case when $c = -\dot{F}(T)/(pq)$ and $a_{R_n}(i) = a_{R_n}(i,f)$, the scores corresponding to the truncated density $f(x)/p$, $x \leq T$.

Henceforth, we assume $\{r_n\}$ is a subsequence of $\{n\}$ such that $r_n/n \to p$ as $n \to \infty$, and for given $\{r_n\}$, the scores $a_{r_n}(i)$ converge in quadratic mean to a square integrable function ϕ with $\int_0^1 (\phi(u)-\overline{\phi})^2 du > 0$ where $\overline{\phi} = \int_0^1 \phi(u)du$. We also assume that

$$\overline{a}_{r_n} \equiv r_n^{-1}\sum_{i=1}^{r_n} a_{r_n}(i) = \overline{\phi} + o(n^{-1/2}), \text{ and } \lambda_n = \lambda + o(n^{-1/2}), \quad 0 < \lambda < 1.$$

Consider a contiguous sequence of location alternatives $\{Q_n\}$ given by the pdf's $q_n = \prod_{i=1}^{n_1} f(x_i) \prod_{i=n_1+1}^{n} f(x_i - \theta_n)$ where $\theta_n = \Delta n^{-1/2}$, $\Delta > 0$, and f is assumed to have finite Fisher information. We will use $\xrightarrow{D_n}$ and $\xrightarrow{D_0}$ to denote convergence in distribution under Q_n and H_0, respectively, and the corresponding symbols for convergence in probability will be $\xrightarrow{Q_n}$ and $\xrightarrow{P_0}$.

The following theorem states the asymptotic distribution of S_n under Q_n. Its proof follows from the result (c) of Lemma A.2 in the Appendix.

<u>Theorem 3.1</u> Let $p_n = F(T-\theta_n)$. Then, under Q_n, the statistic S_n is asymptotically $N(\nu_n, \sigma^2)$ with

$$\nu_n = (1-\lambda)p_n\bar{\phi} + \Delta n^{-1/2}\lambda(1-\lambda)p \int_0^1 \phi(u)\phi(u,f)du + (p-p_n)\lambda_n(1-\lambda_n)c,$$

$$\sigma^2 = \lambda(1-\lambda)p \int_0^1 (\phi(u)-\bar{\phi})^2 du + (1-\lambda)pq\bar{\phi}^2 + \lambda(1-\lambda)pqc^2 - 2\lambda(1-\lambda)pqc\bar{\phi},$$

where $\phi(u,f)$ is given by (2.5).//

In order to derive the asymptotic power of the LMPS test we first construct an allied unconditional test which is based on the same test statistic, has asymptotic level α, and moreover, is asymptotically power-equivalent to the (conditional) LMPS test. The work is thereby reduced to the evaluation of the asymptotic power of the allied test. The required result is stated in Theorem 3.2 and is proved in Appendix B.

<u>Theorem 3.2</u> Suppose that $\hat{\mu}_n$ and $\hat{\sigma}_n$ (either constants or functions of $\bar{R}_n$) are chosen such that the statistic $S_{nA} \equiv n^{1/2}(S_n - \hat{\mu}_n)/\hat{\sigma}_n$ satisfies

(a) $S_{nA} \xrightarrow{D_0} N(0,1)$

(b) $S_{nA}|_{\bar{R}_n=\bar{r}_n} \xrightarrow{D_0} N(0,1)$ as $\bar{r}_n \to p$.

Then the unconditional test given by $\chi_A = 1(0)$ if $S_{nA} \geq (<) z_\alpha$ is of asymptotic level α. Moreover, if

(c) S_{nA} has a limiting distribution under Q_n,

then the test χ_A is asymptotically power-equivalent to the conditional test $\chi = 1$, $\gamma(\bar{r}_n)$ or 0, if $S_{nA} >, =, < b_\alpha(\bar{r}_n)$ with $\gamma(\bar{r}_n)$ and $b_\alpha(\bar{r}_n)$ determined by

$$\alpha = P_0[S_{nA} > b_\alpha(\bar{r}_n)|\bar{R}_n=\bar{r}_n] + \gamma(\bar{r}_n)P_0[S_{nA} = b_\alpha(\bar{r}_n)|\bar{R}_n=\bar{r}_n]. \tag{3.2}$$

Note that S_{nA} is a function of R_n and S_n. Hence the conditional test χ based on S_{nA} is equivalent to the LMPS test. This theorem states that we can use χ_A to determine the asymptotic power of χ once we find $\hat{\mu}_n$ and $\hat{\sigma}_n$ such that conditions (a) - (c) are satisfied.

To verify conditions (a) and (b) of Theorem 3.2, we first consider the joint limiting distribution of S_n and $\bar{R}_n$ under H_0. This distribution can be obtained by using $\Delta = 0$ and $p_n = p$ in part (c) of Lemma A.2. Consequently, we have

$$n^{1/2}[S_n-(1-\lambda)p\bar{\phi}, \bar{R}_n-p] \xrightarrow{D_0} N(\underset{\sim}{0}, \underset{\sim}{\Sigma}_2) \tag{3.3}$$

where $\underset{\sim}{\Sigma}_2$ is given by (A.5). In view of the fact that $\phi(u) = \phi(u,f)$ for the LMPS test, we have here $\bar{\phi} = 0$. It then follows from (A.5), (A.6) and (3.3) that

$$n^{1/2}S_n/\sigma_0 \xrightarrow{D_0} N(0,1) \tag{3.4}$$

and as $\bar{r}_n \to p$

$$n^{1/2}S_n/\sigma_0|_{\bar{r}_n} \xrightarrow{D_0} N(0,1) \tag{3.5}$$

where

$$\sigma_0^2 \quad = \quad \lambda(1-\lambda)p[\int_0^1 \phi^2(u)du + qc^2]. \qquad (3.6)$$

Now (3.3) and (3.4) entail that a statistic S_{nA} can be defined to satisfy conditions (a) and (b) of Theorem 3.2 by setting $\hat{\mu}_n = 0$ and having $\hat{\sigma}_n \xrightarrow{P_0} \sigma_0$. To obtain such a $\hat{\sigma}_n$, we can simply replace p by $\bar{R}_n$ in (3.6). Note that $\int_0^1 \phi^2(u)du$ may contain p, for example, when $\phi(u) = \phi(u,f)$ of (2.5). Denoting $\int_0^1 \phi^2(u)du$ or its estimate, whichever applies, by V, we get

$$\hat{\sigma}_n^2 \quad = \quad \lambda(1-\lambda)\bar{R}_n[V + (1-\bar{R}_n)c^2]. \qquad (3.7)$$

To ensure $\hat{\sigma}_n \xrightarrow{P_0} \sigma_0$, we will confine ourselves to the situations where $\int_0^1 \phi^2(u)du$ is continuous in p, if p is involved.

For condition (c) of Theorem 3.2, we need to show that the statistic $S_{nA} = n^{1/2}S_n/\hat{\sigma}_n$ with $\hat{\sigma}_n^2$ given by (3.7), possesses a limiting distribution under the sequence $\{Q_n\}$ of contiguous location alternatives. To this end, we first use $\bar{\phi} = 0$ into the assertions of Theorem 3.1 and obtain that under Q_n, $n^{1/2}(S_n-\mu_{0n})/\sigma_0$ is asymptotically N(0,1) with

$$\mu_{0n} \quad = \quad \Delta n^{-1/2}\lambda(1-\lambda)\ p\int_0^1 \phi(u)\phi(u,f)du + (p-p_n)\lambda_n(1-\lambda_n)c$$

and σ_0^2 given by (3.6). However since $\hat{\sigma}_n \xrightarrow{P_0} \sigma_0$, we have $\hat{\sigma}_n \xrightarrow{Q_n} \sigma_0$ by contiguity. Thus $n^{1/2}(S_n-\mu_{0n})/\hat{\sigma}_n$, or equivalently, $S_{nA} - n^{1/2}\mu_{0n}/\hat{\sigma}_n$, is asymptotically N(0,1) under Q_n. Moreover since $n^{1/2}(p-p_n) \to \Delta f(T)$, we have $n^{1/2}\mu_{0n}/\hat{\sigma}_n \xrightarrow{Q_n} \mu(\Delta)/\sigma_0$ and in turn $S_{nA}-[\mu(\Delta)/\sigma_0] \xrightarrow{D_0} N(0,1)$ where

$$\mu(\Delta) \quad = \quad \lambda(1-\lambda)\ \Delta\ [p\int_0^1 \phi(u)\phi(u,f)du + f(T)c].$$

This finally entails that the asymptotic power of χ_A and therefore of χ is given by

$$\lim_{n\to\infty} Q_n(S_{nA} \geq z_\alpha) = 1-\Phi(z_\alpha-\mu(\Delta)/\sigma_0)$$
$$= 1-\Phi(z_\alpha-k\Delta)$$

where $\Phi(\cdot)$ denotes the cdf of $N(0,1)$ and

$$k^2 = \lambda(1-\lambda)[p \int_0^1 \phi(u)\phi(u,f)du + f(T)c]^2[p(\int_0^1 \phi^2(u)du + qc^2)]^{-1}. \quad (3.8)$$

The number k is the efficacy of the test corresponding to the density f.

Remark. The preceding arguments also apply to contiguous scale alternatives in which case we need to replace $f(T)$ by $Tf(T)$ and $\phi(u,f)$ by $\phi_1(u,f)$ of (2.7).

4. ASYMPTOTIC RELATIVE EFFICIENCY

Drawing from the results of the previous section we now proceed to compare Halperin's test with the LMPS test S_L given by (2.8) in terms of their (Pitman) asymptotic relative efficiency (AE). In addition, we will also consider the AE of S_L vs another modification of the WMW test proposed by Gehan (1965). Both AE's are established under the logistic location model.

Gehan's statistic, say W_G, was originally constructed in the context of arbitrary right censoring. For the special case of equal type I censoring, W_G is related to Halperin's statistic W_H by

$$W_G = 2W_H + \frac{1}{n^2}[R_1R_2-n_2R_1-n_1R_2].$$

The difference between W_H and W_G arises from the fact that while W_H includes the information provided by the ordering of the uncensored observations as well as the uncensored X's with the censored Y's, W_G additionally takes into account the contribution of the uncensored Y's and censored X's.

Under the logistic location model, we have $\phi(u,f) = p(2u-1)$, $p = [1+\exp(-T)]^{-1}$ and $f(T) = \exp(-T)[1+\exp(-T)]^{-2}$. The statistic S_L is a special case of (3.1) with $\phi(u) = \phi(u,f)$ and $c = 1$. Using these into (3.8), we obtain the square efficacy of S_L as

$$k_{S_L}^2 = \frac{\lambda(1-\lambda)[p^3/3 + f(T)]^2}{p(p^2/3 + q)} .$$

The asymptotics of W_H and W_G can be derived along the same lines; the details are therefore omitted. It turns out that the square efficacy of the conditional test based on W_H is

$$k_{W_H}^2 = \lambda(1-\lambda)\{p^2 \int_0^1 (2u-1)\phi(u,f)du + f(T)[p+2q(1-\lambda)]\}^2$$
$$\times \{p[p^2/3 + q((1-\lambda)(2-p)^2 + \lambda p^2 - 4\lambda(1-\lambda)q^2)]\}^{-1} .$$

Under the logistic model, the AE of W_H relative to S_L is therefore given by

$$\begin{aligned} AE(W_H:S_L) &= k_{W_H}^2/k_{S_L}^2 \\ &= (p^2/3 + q)\{p^3/3 + f(T)[p+2q(1-\lambda)]\}^2[p^3/3 + f(T)]^{-2} \\ &\quad \times \{p^2/3 + q[(1-\lambda)(2-p)^2 + \lambda p^2 - 4\lambda(1-\lambda)q^2]\}^{-1} . \end{aligned}$$

Numerical computations of the above AE for different T and λ are presented in Table 4.1. Numbers are accurate up to 6 decimal places. The AE is exactly 1 at $\lambda = 1/2$ so Halperin's test is asymptotically optimal when the sample sizes are equal. Furthermore, all AE's are close to 1, and they tend to 1 as $\lambda \to 1/2$ or as $p \to 1$ or 0.

Table 4.1

Values of AE (W_H:S_L) under the logistic location model.

λ \ T	-3.0	-1.0	0	1.0	3.0
p	.047428	.268941	.500000	.731058	.952574
.1	.999854	.995709	.989252	.991378	.999833
.2	.999896	.997080	.993075	.994842	.999906
.3	.999942	.998397	.996444	.997560	.999958
.4	.999980	.999494	.998963	.999350	.999990
.5	1.000000	1.000000	1.000000	1.000000	1.000000
.6	.999960	.999104	.998537	.999260	.999989
.7	.999704	.994885	.992916	.996839	.999958
.8	.998607	.982354	.980421	.992403	.999904
.9	.992043	.959849	.956601	.985572	.999830

For the conditional test with the test statistic W_G, the square efficacy is given by

$$k^2_{W_G} = \lambda(1-\lambda)[p^2 \int_0^1 (2u-1)\phi(u,f)du+f(T)]^2[p(p^2/3 +q)]^{-1}.$$

In the special case when $\phi(u,f) = p(2u-1)$, we find that $k^2_{W_G} = k^2_{S_L}$ so $AE(W_G:S_L) = 1$. Therefore, Gehan's test is asymptotically optimal, although it does not have the LMPS property in finite samples.

5. CONCLUDING REMARKS

The idea of LMPS rank test can be extended to the unequal as well as multiple type I censoring cases.

Under unequal type I censoring, each sample has its own specified censoring point, say T_i, $i = 1,2$. For this setting, ordering is only possible for observations that are $\leqslant T$ where $T = \min(T_1,T_2)$. Thus, the censored rank-order $\underset{\sim}{Z}_c$ is still equal to $(Z_1,Z_2,\cdots,Z_R)$, where R is the total number of X_i's and Y_j's that are $\leqslant T$. Consequently, the unequal type I censoring can be treated in the same way as the equal type I censoring.

As to the multiple type I censoring, we consider the case of double type I censoring as an example. Let T_L denote the maximum of the left censoring times and T_U the minimum of the right censoring times of the two samples and assume that $T_L < T_U$. Let R_L denote the total number of observations that are $\leq T_L$ and R_B, the total number of observations that occur in the interval (T_L, T_U). Then, the censored rank-order vector is given by $\underset{\sim}{Z}_c = (Z_{R_L+1}, \cdots, Z_{R_L+R_B})$, and

$$P_{H_0}(\underset{\sim}{z}_c) = kF^{r_L}(T_L)[F(T_U)-F(T_L)]^{r_B}[1-F(T_U)]^{n-r_L-r_B}$$

where k is a constant free of F. Here the sufficient statistics (R_L, R_B) have a trinomial distribution under H_0, and are also complete. Thus we can derive the LMPS rank tests using the conditional likelihood of $\underset{\sim}{Z}_c$ given (R_L, R_B).

APPENDIX A: PROOF OF THEOREM 3.1

A theorem due to Sethuraman (1961) concerning conditional and joint limiting distributions provides the basic tool for a fairly straightforward treatment of the asymptotics of the conditional tests.

Theorem A.1 [Sethuraman] Let $\{\underset{\sim}{\varepsilon}_n\}$ and $\{\underset{\sim}{\eta}_n\}$ be sequences of random ℓ- and m-vectors defined on a probability space. If, for an arbitrary $\underset{\sim}{t} \in R^m$, the conditional distribution of $\underset{\sim}{\varepsilon}_n$, given $\underset{\sim}{\eta}_n = \underset{\sim}{t}$, converges to $N_\ell(\underset{\sim}{B}\underset{\sim}{t},\underset{\sim}{\Gamma})$, and $\underset{\sim}{\eta}_n$ strongly converges in distribution (SD) to $N_m(\underset{\sim}{0},\underset{\sim}{\Omega})$, then jointly $(\underset{\sim}{\varepsilon}_n,\underset{\sim}{\eta}_n)$ converges in distribution to $N_{\ell+m}(\underset{\sim}{0},\underset{\sim}{\Sigma})$ with

$$\underset{\sim}{\Sigma} = \begin{bmatrix} \underset{\sim}{\Gamma}+\underset{\sim}{B}\underset{\sim}{\Omega}\underset{\sim}{B}' & , & \underset{\sim}{B}\underset{\sim}{\Omega} \\ \underset{\sim}{\Omega}'\underset{\sim}{B}' & , & \underset{\sim}{\Omega} \end{bmatrix} .$$

We decompose S_n of (3.1) as $S_n = S_{1n}+S_{2n}$ with

$$S_{1n} = c\lambda_n(1-\lambda_n)(\bar{R}_{1n}-\bar{R}_{2n}), \quad S_{2n} = n^{-1}\sum_{i=1}^{R_n}(1-Z_i)a_{R_n}(i).$$

To derive the asymptotic distribution of S_n, we first obtain the conditional limiting distribution of S_{2n} given R_{1n} and R_{2n}. An application of Theorem A.1 will then give the joint limiting distribution of S_{2n}, R_{1n} and R_{2n}. Finally, Theorem 3.1 will be established using an appropriate linear transformation of these variables.

Lemma A.1 Let $\{r_{1n}\}$ and $\{r_{2n}\}$ be subsequences of $\{n\}$ such that $r_{1n}/n \to \lambda p$ and $r_{2n}/n \to (1-\lambda)p$. Denote

$$r_n = r_{1n}+r_{2n}$$

$$\nu_{1n} = \frac{\Delta}{\sqrt{n}}\frac{r_{1n}r_{2n}}{nr_n}\int_0^1 \phi(u)\phi(u,f)du \tag{A.1}$$

$$\sigma_{1n}^2 = \frac{r_{1n}r_{2n}}{nr_n}\int_0^1 (\phi(u)-\bar{\phi})^2du. \tag{A.2}$$

Then conditionally given $R_{1n} = r_{1n}$ and $R_{2n} = r_{2n}$, the limiting distribution of $n^{1/2}\{[S_{2n}-(r_{2n}/n)\bar{a}_{r_n}]-\nu_{1n}\}/\sigma_{1n}$ under Q_n is $N(0,1)$.

Proof: Under Q_n and given $R_{1n} = r_{1n}$ and $R_{2n} = r_{2n}$, $X_{(1)},\cdots,X_{(r_{1n})}$ and $Y_{(1)},\cdots,Y_{(r_{2n})}$ are simply the order statistics of independent random samples of sizes r_{1n} and r_{2n}, respectively, from the truncated distribution $h(x) = f(x)/F(T)$ and $h_{\theta_n}(x) = f(x-\theta_n)/F(T-\theta_n)$, $x \leq T$. Hence, given (r_{1n},r_{2n}), the rank-order vector $\underset{\sim}{Z}_c = (Z_1,\cdots,Z_{r_n})$ has the same probability distribution as the distribution of the rank-order vector corresponding to two uncensored independent random samples of sizes r_{1n} and r_{2n} from $h(x)$ and $h_{\theta_n}(x)$, respectively. Since $S_{2n} = n^{-1}\sum_{i=1}^{r_n}(1-Z_i)a_{r_n}(i)$ is a linear rank statistic of the full rank order $\underset{\sim}{Z}_c$ in this context, we can apply Theorem VI.2.3 of Hájek and Šidák (1967) to establish the conditional asymptotic distribution. Using $d_i = 0$, $1 \leq i \leq r_{1n}$ and $d_i = \Delta n^{-1/2}$, $r_{1n}+1 \leq i \leq r_n$, in that theorem, the expressions (A.1) and (A.2) are obtained.//

Next, we define $\eta_{1n} = n^{1/2}(\bar{R}_{1n}-p)$ and $\eta_{2n} = n^{1/2}(\bar{R}_{2n}-p_n)$. Since R_{1n} and R_{2n} have binomial distributions $b(n_1,p)$ and $b(n_2,p_n)$, respectively, and $p_n \to p$ by continuity of F, we conclude that under Q_n, $\eta_{1n} \xrightarrow{SD} N(0,pq)$ $\eta_{2n} \xrightarrow{SD} N(0,pq)$. These, together with the facts that R_{1n} and R_{2n} are independent and $\lim_{n\to\infty} n_1/n = \lambda$ and $\lim_{n\to\infty} n_2/n = 1-\lambda$, yield that under Q_n

$$[\eta_{1n},\eta_{2n}] \xrightarrow{SD} N(\underset{\sim}{0},\underset{\sim}{\Omega}) \tag{A.3}$$

where

$$\underset{\sim}{\Omega} = \begin{bmatrix} \lambda^{-1}pq & , & 0 \\ 0 & , & (1-\lambda)^{-1}pq \end{bmatrix}. \tag{A.4}$$

Lemma A.2. Under Q_n, we have

$$\text{(a)}\quad n^{1/2}(S_{2n}-\nu_{2n})\Big|_{\eta_{1n}=x,\,\eta_{2n}=y} \xrightarrow{D_n} N((1-\lambda)\bar{\phi}y,\sigma_1^2)$$

$$\text{(b)}\quad n^{1/2}[S_{2n}-\nu_{2n},\bar{R}_{1n}-p,\bar{R}_{2n}-p_n] \xrightarrow{D_n} N(\underset{\sim}{0},\underset{\sim}{\Sigma}_1)$$

$$\text{(c)}\quad n^{1/2}[S_n-\nu_n,\bar{R}_n-(p+(1-\lambda_n)(p_n-p))] \xrightarrow{D_n} N(\underset{\sim}{0},\underset{\sim}{\Sigma}_2)$$

where

$$\nu_{2n} = (1-\lambda)p_n\bar{\phi} + \frac{\Delta}{\sqrt{n}}\lambda(1-\lambda)p\int_0^1 \phi(u)\phi(u,f)du$$

$$\nu_n = \nu_{2n} + (p-p_n)\lambda_n(1-\lambda_n)c$$

$$\underset{\sim}{\Sigma}_1 = \begin{bmatrix} \sigma_1^2+(1-\lambda)pq\bar{\phi}^2, & 0, & pq\bar{\phi} \\ 0, & \lambda^{-1}pq, & 0 \\ pq\bar{\phi}, & 0, & (1-\lambda)^{-1}pq \end{bmatrix},$$

$$\underset{\sim}{\Sigma}_2 = \begin{bmatrix} \sigma^2, & (1-\lambda)pq\bar{\phi} \\ (1-\lambda)pq\bar{\phi}, & pq \end{bmatrix} \tag{A.5}$$

with

$$\sigma_1^2 = \lambda(1-\lambda)p\int_0^1 (\phi(u)-\bar{\phi})^2du$$

$$\sigma^2 = \sigma_1^2+(1-\lambda)pq\bar{\phi}^2 + \lambda(1-\lambda)pqc^2 - 2\lambda(1-\lambda)pqc\bar{\phi}\,. \tag{A.6}$$

Proof: First note that the conditions $\eta_{1n} = x$ and $\eta_{2n} = y$ are essentially the same as $R_{1n} = [r_{1n}]$ and $R_{2n} = [r_{2n}]$ with $r_{1n} = n_1(x/\sqrt{n} + p)$ and $r_{2n} = n_2(y/\sqrt{n} + p_n)$. We have $r_{1n}/n \to \lambda p$, $r_{2n}/n \to (1-\lambda)p$. Therefore from (A.1) and (A.2) we obtain $\lim_{n\to\infty} \sigma_{1n}^2 = \lambda(1-\lambda)p\int_0^1 (\phi(u)-\bar{\phi})du = \sigma_1^2$ and

$$n^{1/2}\left(\frac{r_{2n}}{n}\bar{a}_{r_n}+\nu_{1n}-\nu_{2n}\right)$$

$$= \frac{n_2}{n} y\,\bar{a}_{r_n} + n^{1/2}\frac{n_2}{n} p_n(\bar{a}_{r_n}-\bar{\phi}) + n^{1/2}\left[\frac{n_2}{n} - (1-\lambda)\right]p_n\bar{\phi}$$

$$+ \Delta\left[\frac{r_{1n}r_{2n}}{nr_n} - \lambda(1-\lambda)p\right]\int_0^1 \phi(u)\phi(u,f)du \longrightarrow (1-\lambda)y\bar{\phi}, \text{ and } n\to\infty.$$

Using these results and Lemma A.1, part (a) is concluded.

The assertion (b) follows from part (a) and (A.3) by using Theorem A.1. Here $\underset{\sim}{\Sigma}_1$ is obtained by substituting σ_1^2 for $\underset{\sim}{\Gamma}$, $\underset{\sim}{B} = (0,(1-\lambda)\bar{\phi})$ and $\underset{\sim}{\Omega}$ given by (A.4) into $\underset{\sim}{\Sigma}$ of Theorem A.1.

For part (c), let us use the representation

$$n^{1/2}[S_n-(\nu_{2n}+(p-p_n)\lambda_n(1-\lambda_n)c),\ \bar{R}_n-(p+(1-\lambda_n)(p_n-p))]$$

$$= \underset{\sim}{A}_n n^{1/2}[S_{2n}-\nu_{2n}, \bar{R}_{1n}-p, \bar{R}_{2n}-p_n]$$

where

$$\underset{\sim}{A}_n = \begin{bmatrix} 1, & \lambda_n(1-\lambda_n)c, & -\lambda_n(1-\lambda_n)c \\ 0, & \lambda_n, & 1-\lambda_n \end{bmatrix}.$$

Since $\underset{\sim}{A}_n$ converges to

$$\underset{\sim}{A} = \begin{bmatrix} 1, & \lambda(1-\lambda)c, & -\lambda(1-\lambda)c \\ 0, & \lambda, & 1-\lambda \end{bmatrix},$$

part (c) follows from part (b) by noting that $\underset{\sim}{\Sigma}_2 = \underset{\sim}{A}\underset{\sim}{\Sigma}_1\underset{\sim}{A}'$.//

APPENDIX B: PROOF OF THEOREM 3.2

The fact that χ_A is of asymptotic level α is obvious from the condition (a). To prove that χ and χ_A are asymptotically power-equivalent, we will use a result due to Hoeffding (1952, p189) concerning the convergence in probability of functions of random variables.

Let

$$g_n(\bar{r}_n) = \gamma(\bar{r}_n) Q_n(S_{nA} = b_\alpha(\bar{r}_n) | \bar{R}_n = \bar{r}_n).$$

Then under Q_n the asymptotic power of χ is given by the limit of

$$E_{\theta_n}[E_{\theta_n}(\chi|\bar{R}_n)] = Q_n(S_{nA} > b_\alpha(\bar{R}_n)) + E_{\theta_n}[g_n(\bar{R}_n)].$$

We need to show that the limit of $E_{\theta_n}[E_{\theta_n}(\chi|\bar{R}_n)]$ equals the limit of $Q_n(S_{nA} > z_\alpha)$, the asymptotic power of χ_A. From condition (b), we have that

$$P_0(S_{nA} = b_\alpha(\bar{r}_n) | \bar{R}_n = \bar{r}_n) \to 0 \text{ as } \bar{r}_n \to p \qquad \text{(B.1)}$$

which together with (3.2) entails that $b_\alpha(\bar{r}_n) \to z_\alpha$ as $\bar{r}_n \to p$. This and the fact $\bar{R}_n \xrightarrow{P_0} p$ imply, by Hoeffding's result, $b_\alpha(\bar{R}_n) \xrightarrow{P_0} z_\alpha$. Further, on account of contiguity, we have $b_\alpha(\bar{R}_n) \xrightarrow{Q_n} z_\alpha$ which in view of condition (c), leads to

$$\lim_{n\to\infty} Q_n(S_{nA} > b_\alpha(\bar{R}_n)) = \lim_{n\to\infty} Q_n(S_{nA} > z_\alpha).$$

The proof will be concluded if we show that $\lim E_{\theta_n}[g_n(\bar{R}_n)] = 0$. Once again, using contiguity we have $\bar{R}_n \xrightarrow{Q_n} p$ and from (B.1) that $Q_n(S_{nA} = b_\alpha(\bar{r}_n) | \bar{R}_n = \bar{r}_n) \to 0$ as $\bar{r}_n \to p$. An application of Hoeffding's result again implies $g_n(\bar{R}_n) \xrightarrow{Q_n} 0$. Thus the unconditional test χ_A is asymptotically power-equivalent to the conditional test χ.

REFERENCES

Bhattacharyya, G. K. and Mehrotra, K. G. (1983), "A Unified Treatment of Locally Most Powerful Rank Tests Under Type II Censoring," Journal of the American Statistical Association, 78, 375-381.

Crowley, J. (1974), "A Note on Some Recent Likelihoods Leading to the Log Rank Test," Biometrika, 61, 533-538.

Cox, D. R. (1972), "Regression Models and Life Tables," Journal of the Royal Statistical Society, Ser. B, 34, 187-220.

Gehan, E. (1965), "A Generalized Wilcoxon Test for Comparing Arbitrarily Singly-Censored Samples," Biometrika, 52, 203-223.

Hájek, J. and Šidák, Z. (1967), Theory of Rank Tests, New York: Academic Press.

Halperin, M. (1960), "Extension of the Wilcoxon-Mann-Whitney Test to Samples Censored at the Same Fixed Point," Journal of the American Statistical Association, 55, 125-138.

Hoeffding, W. (1952), "The Large-Sample Power of Tests Based on Permutations of Observations," Annals of Mathematical Statistics, 23, 169-192.

Johnson, R. A. and Mehrotra, K. G. (1972), "Locally Most Powerful Rank Tests for the Two-Sample Problem with Censored Data," Annals of Mathematical Statistics, 43, 823-831.

Kalbfleisch, J. D. and Prentice, R. L. (1973), "Marginal Likelihoods Based on Cox's Regression and Life Model," Biometrika, 60, 267-278.

Peto, R. and Peto, J. (1972), "Asymptotically Efficient Rank Invariant Test Procedures," Journal of the Royal Statistical Society, Ser. A, 135, 185-206.

Saleh, A. K. Md. Ehsanes, Scott, C., and Junkins, D. B. (1975), "Exact First and Second Order Moments of Order Statistics from the Truncated Exponential Distribution," Naval Research Logistics Quarterly, 22, 65-77.

Sethuraman, J. (1961), "Some Limit Theorems for Joint Distributions," Sankhyā A, 23, 379-386.

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ABSTRACT

Locally most powerful similar (LMPS) rank tests are derived for the two-sample problem when the samples are type I censored on the right. Special parametric families are then considered to arrive at the censored-sample modifications of the Wilcoxon-Mann-Whitney (WMW) test, the Savage test and the median test from the point of view of LMPS as an exact optimality criterion. Asymptotic power function of the LMPS test is derived under a sequence of contiguous alternatives, and the censored sample modifications of the WMW test, due to Halperin (1960) and Gehan (1965), are compared to the LMPS test in terms of asymptotic efficiency. It is found that although Gehan's test is not exactly equivalent to the LMPS test, it is in fact asymptotically optimal, whereas Halperin's test is not even asymptotically optimal for the logistic location model.