CURVES 2

LAST TIME: Properties of Curves
THIS TIME: MAKING CURVES FROM PIECES

What kinds of pieces?
Polynomials - line segments
cubics ← why?

Polynomial

$$a_0 v^0 + a_1 v^1 + a_2 v^2 + a_3 v^3 \ldots$$

linear combination of blending functions

$$a_0 b_0(v) + a_1 b_1(v) \ldots$$

a_n = controls

simplest blending functions

$b_0(v) = 1$ $\quad b_2(v) = v^2$ $\quad \ldots$

$b_1(v) = v$

Polynomials of degree n are a vector space
Set of functions are a basis ← just like vectors

Matrix Form

$$\begin{bmatrix} 1 & v & v^2 & v^3 \ldots \end{bmatrix} \begin{bmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_n \end{bmatrix}$$

different basis matrices = different sets of blending functions
= different controls

Example: LINE SEGMENTS

$f_x = a_{0x} + a_{1x}(v)$
$f_y = a_{0y} + a_{1y}(v)$
$b_0 = 1 \quad b_1 = v$

↪ just consider x, or use vector notation

a_0 and a_1 aren't convenient controls

want p_0, p_1 = end points

new blending functions:

$b_0 = (1 - v)$
$b_1 = v$

$f(v) = p_0 b_0(v) + p_1 b_1(v)$

overkill for line segments
better for harder things

Deriving the basis matrix / blending functions

write constraints

$p_0 = f(0)$ @ $v = 0$
$p_1 = f(1)$ @ $v = 1$

convert $p \rightarrow a$

$p_0 = a_0 + 0 \, a_1$
$p_1 = a_0 + 1 \, a_1$

$p = C a$

$$C = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$a = C^{-1} p$

$a = B p$

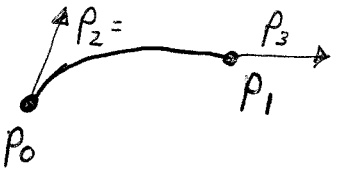
$f(v) = v B p$
$b(v) = v B$

Cubics

$$a_0 + a_1 v + a_2 v^2 + a_3 v^3$$

Really inconvenient!

Hermite form:



easy for make $C(1)$

not same as book!

$$p_0 = f(0) = a_0 + a_1 v^0 + a_2 v^0 + a_3 v^0$$
$$p_1 = f(1) = a_0 + a_1 v^1 + a_2 v^1 + a_3 v^1$$
$$p_2 = f'(0) = a_1 + 2a_2 v^0 + 3a_3 v^0$$
$$p_3 = f'(1) = a_1 + 2a_2 v^1 + 3a_3 v^1$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix} \qquad B = C^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix}$$

$$b_1(v) = 1 - 3v^2 + 2v^3$$
$$b_2(v) = 3v^2 - 2v^3$$
$$b_3(v) = v - 2v^2 + v^3$$
$$b_4(v) = -v^2 + v^3$$

Check to see interpolation