Sept 13, 2001

559 DAY 4

Programs due today
2 surveys for books (others) (4 others w/o survey)

Next weeks program

PAUSCH UPL time

Today SAMPLING, RE-SAMPLING, AND
What is a Pixel Anyway

Hanrahan's notes - better than mine, albeit more formal

Problems:

SAMPLING continuous → discrete
RECONSTRUCTION discrete → continuous
RE-SAMPLING discrete → discrete

SAMPLING:

DON'T Know What happens in between

WITHOUT ADDITIONAL INFO

What additional info?] need language to talk about signals

9-13-2001 PL

FOURIER SERIES

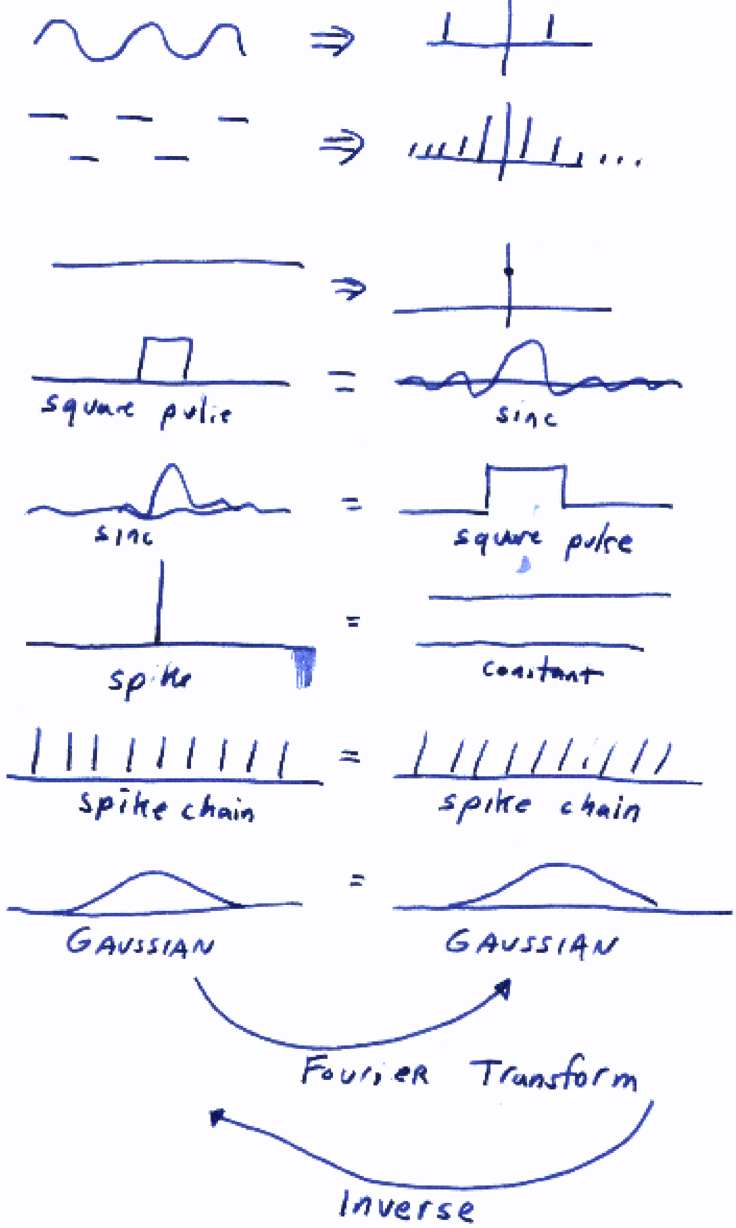
Orthogonality
Different Domains
Time   Wavelet
Fourier
Gabor

add up sines and cosines ...

way to talk about how things change

? HF   LF

different view of same thing



Can you transform an operator?

Multiplication ⇒ CONVOLUTION

CONVOLUTION

see p5 of last years NOTES

BACK TO SAMPLING

No Turnarounds =
No HF

$\omega/2$
← Nyquist LIMIT

See p3

What does this mean for us?

A little blurring is not neccessarily bad
- don't really know where sharp edges are

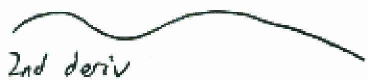
PRE-FILTER only way to sample w/ guarantees

RE-SAMPLING
Blur before reduce ← filter, convolution
Interpolate to enlarge
Wierd stretching of image?

Sampling - check discrete times  ≡ might miss something in between

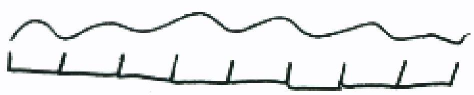
uniform sampling vs. non-uniform

reconstruction -
make continuous

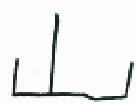
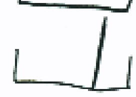
lots of possible ways ← draw
smoothly interpolate 
cubic, minimum 2nd deriv
linear interpolate 

spike chain 

Better Sampling?
Average region of sample? - This is how eye works
How CCD works

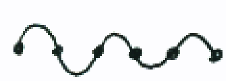
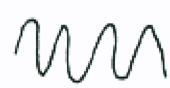


still lose fine details
but they have effect
lack of sharpness

can't tell  from  or 

still sampling - albeit "better" sampling

General Problem -
If things change too fast, miss them

 or  or  or 

Observation -

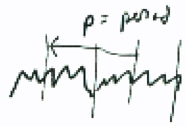
if no rapid "turn around" or abrupt change in direction, we get basic shape

As long as < 1 change per sample, smooth is decent observation

need a ~~better~~ different way to look at signals, in terms of changes

Frequency Domain

periodic signals



$f(t) = f(t+p) = f(t+2p) \ldots$

$\text{frequency} = \frac{1}{p}$

sine waves

why? Fourier Theorem

signals viewed as sum of sine waves

combination = ~~different~~ way to describe signal

Time Domain = how much of each pulse

Fourier Domain = how much of each sine wave

Gabor Domain = ~~m~~ how much of each Gabor chirp

Frequency - low freq = smooth
high = sharp edges

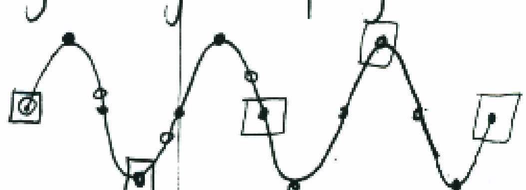
Back to sampling NYQUIST

What is minimum freq sine than can "turn around" inside samples

$1/2 f_{samp}$ - need to sample at twice frequency

Anything faster than this will appear as a lower freq

Aliasing! high frequency looks like lower one



period = 4 f = 1/4

○ sample close to Nyquist - still see oscilations [s<2]

sample at Nyquist - get constant

□ 3/4 F - looks like half speed

NYQUIST LIMIT -

if signal is band limited (no fourier terms above =) to ½ samp F

samp F ≥ 2 top signal F

No Aliasing

Ideal Reconstruction Possible

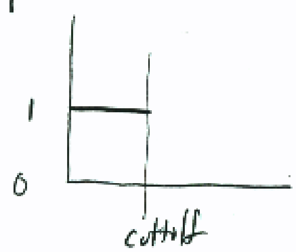
What does a band limited signal look like?

no sharp edges, no quick turn arounds

Once you sample, info is lost

Filter out high frequencies before sampling

Frequency Response



low pass

Boxcar

how to do this?

signal $\xrightarrow{\text{trans}}$ signal mult response

filt signal $\xleftarrow{\text{i trans}}$ filt signal

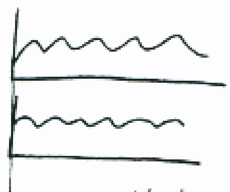
signal $\xleftarrow{\text{i trans}}$ mult response

filt signal

What's the inverse transform of an operator?
(yes, you can do it)

convolution

Convolution – CONVOLUTION!



slide signal fin...

g(0) = multiply together
integrate

Discrete

1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
$\boxed{1\ 2\ 3}$

0= multiply, add
1= shift by 1, mult add
1 2 3
2= 1 2 3

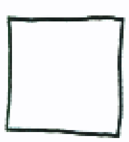
one signal slides

notice = could be zero centered
$\boxed{1\ 2\ 3}$
$\boxed{1\ \boxed{2}\ 3}$ (just a shift of result)

look at $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$
running average

what is an LPF ← examples
sinc

2D convolution ← slide images



seperable filters = can do 1 dimension than others

ideal LPF
Gaussian
Binomial

Binomial filtes
unit box

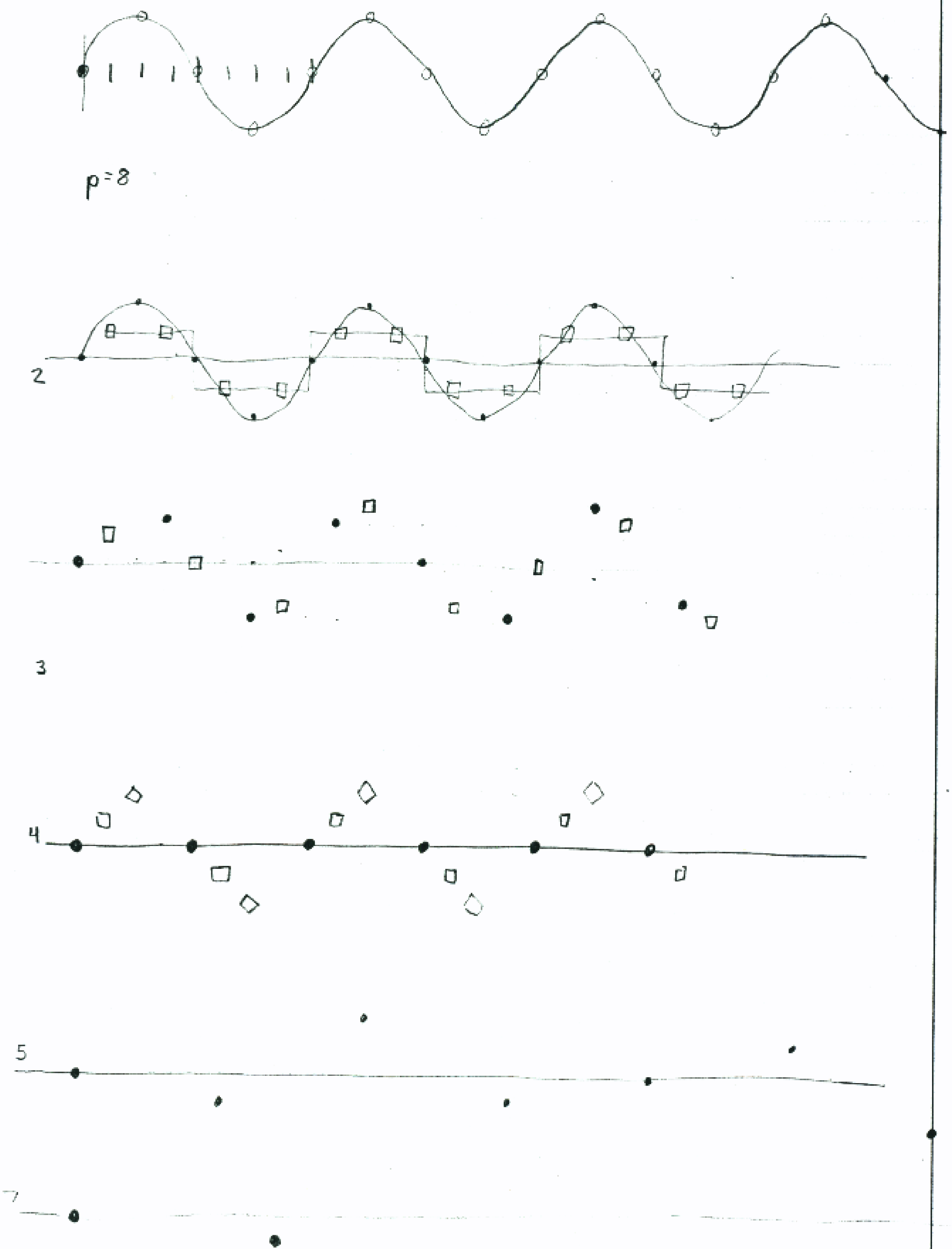
1 1
1 2 1
1 3 3 1
1 4 6 4 1

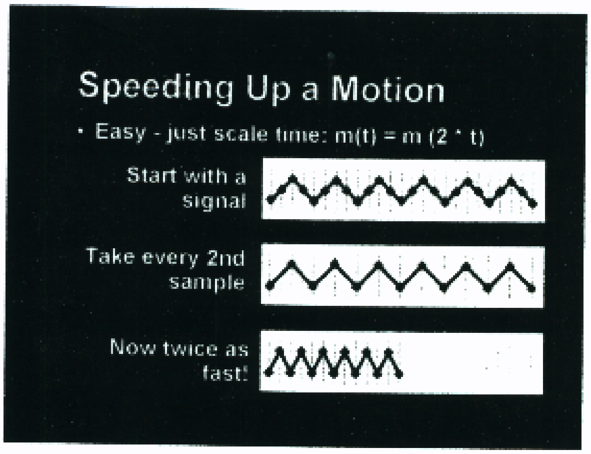
1 1
1 1

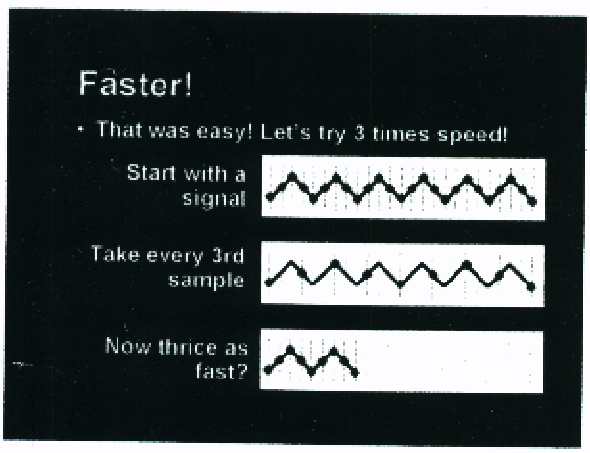
1 2 1
2 4 2
1 2 1

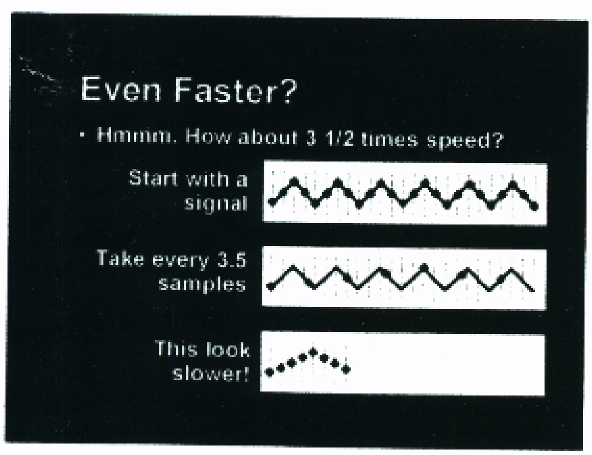
1 3 3 1
3 9 9 3
3 9 9 3
1 3 3 1

Resampling




Speeding Up a Motion
• Easy - just scale time: m(t) = m (2 * t)
Start with a signal
Take every 2nd sample
Now twice as fast!


Faster!
• That was easy! Let's try 3 times speed!
Start with a signal
Take every 3rd sample
Now thrice as fast?


Even Faster?
• Hmmm. How about 3 1/2 times speed?
Start with a signal
Take every 3.5 samples
This look slower!