CS 559: Computer Graphics

Homework 2

This homework must be done individually. Submission date is Tuesday, September 28, 2004, in class.

Question 1:

The $L^*a^*b^*$ color space (defined below) is approximately perceptually uniform. To get from RGB to $L^*a^*b^*$, you first need to get to XYZ, and you also need the *white point* for XYZ, which is the coordinates of white. This question contains five parts.

a. To get from RGB to XYZ, use the matrix given in class. What are the XYZ coordinates for RGB $(1, 0, 0)$?

b. The white point is represented by (X_n, Y_n, Z_n). It is given in terms of $x_n = \frac{X_n}{(X_n+Y_n+Z_n)}$ and $y_n = \frac{Y_n}{(X_n+Y_n+Z_n)}$. Specifically, $x_n = 0.312713$ and $y_n = 0.329016$. Using this information, compute (X_n, Y_n, Z_n), assuming $Y_n = 1.0$. The reading on color contains help for this.

To compute a color in $L^*a^*b^*$, we use the following equations:

$$
\begin{aligned}
L^* &= \begin{cases} 116 * \left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} - 16.0 & \text{if } \frac{Y}{Y_n} > 0.008856 \\ 903.3 * \frac{Y}{Y_n} & \text{otherwise} \end{cases} \\
a^* &= 500\left(f(\frac{X}{X_n}) - f(\frac{Y}{Y_n})\right) \\
b^* &= 200\left(f(\frac{Y}{Y_n}) - f(\frac{Z}{Z_n})\right)
\end{aligned}
$$

where

$$
f(t) = \begin{cases} t^{\frac{1}{3}} & \text{if } t > 0.00856 \\ 7.787t + \frac{16}{116} & \text{otherwise} \end{cases}
$$

Note that when $r = g = b$, all of the ratios (for instance $\frac{Y}{Y_n}$) are approximately equal, and hence a^* and b^* are zero. In other words, the L^* component encodes intensity. You might like to write a program to do the color conversions for the following questions, and test it (testing on white, black and gray is a good start.) You can submit your program if you like.

Colors are frequently linearly interpolated in graphics, as we will see later with *Gourand* shading. That is, to obtain colors between (r_1, g_1, b_1) and (r_2, g_2, b_2), we use the formula

$$
\begin{aligned}
r &= u\, r_1 + (1-u)r_2 \\
g &= u\, g_1 + (1-u)g_2 \\
b &= u\, b_1 + (1-u)b_2
\end{aligned}
$$

where u varies from 0 to 1.

c. Linearly interpolate from (0.5, 0, 0) to (1, 1, 1) in 5 steps. That is, evaluate the formula above for $u \in \{0, 0.25, 0.5, 0.75, 1\}$. Give the 5 resulting RGB colors.

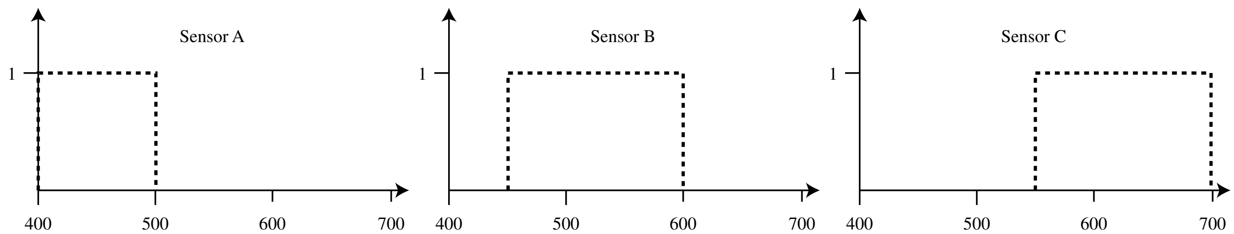
d. Convert each of the colors you obtained n part (c) into (L^*, a^*, b^*). Give the five resulting (L^*, a^*, b^*) coordinates.

e. Plot three graphs: one showing L^* as a function of u, one showing a^* and one showing b^*.

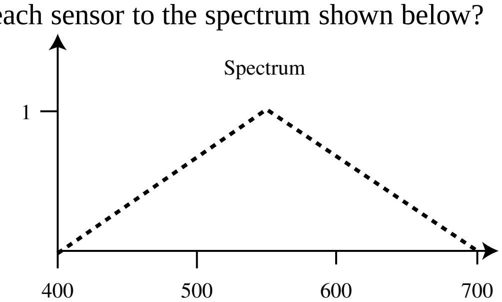
Your graphs should not be straight lines. In other words, linear interpolation in RGB does not give linear interpolation in (L^*, a^*, b^*). To interpolate in a perceptually linear manner, we would interpolate in (L^*, a^*, b^*) space and then convert back into RGB. This is not done for speed reasons.

Question 2:

Consider the three sensors, A, B and C, shown below. Sensor A has a response of 1 between 400nm and 500nm, Sensor B responds between 450nm and 600nm, and Sensor C responds between 550nm and 700nm.

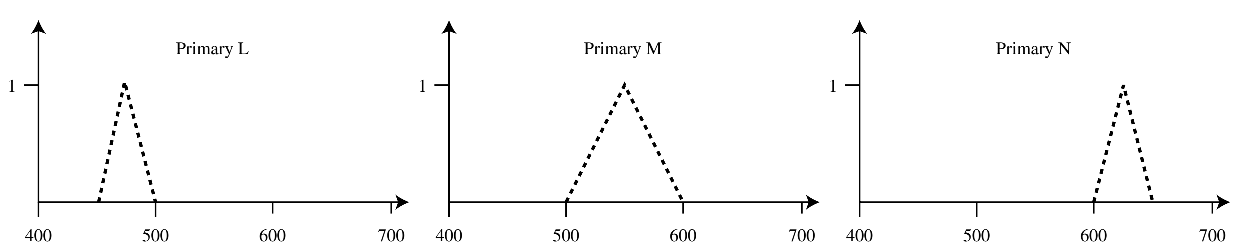


a. What is the response of each sensor to the spectrum shown below?



Your answer should consist of three numbers: the response for A, B and C respectively.

Now you have three primaries, L, M, and N. Primary L emits energy between 450nm and 500nm, with a triangular spectrum. Primary M emits between 500nm and 600nm. Primary L emits between 600nm and 650nm. Each graph shows the output for “1 unit” of primary.



b. What is the response of Sensor B as a function of l, the amount of primary L? You should give an equation that computes $r_{B,L}$ based on l.

c. What is the total response of sensor B resulting from a combination of primaries L, M and N? To answer this, you need to come up with equations for M and N like those in part (b), and then add them together to get an equation that computes r_B as a function of l, m and n.

d. What is the total response of sensors A and C as functions of l, m and n? In other words, repeat parts (b) and (c), but for the other sensors.

e. How much of each primary would be required to simultaneously generate a response from each sensor that is the same as their response to the spectrum of part (a)? To answer this, write down three equations relating the target response from part (a) to l, m and n. Then solve the three equations to find the three unknowns: l, m and n.

This question demonstrates that with three sensors you need three primaries. The primaries have to be linearly independent in the sense that the response of the sensors to one primary cannot be a linear combination of their responses to the others. The color matching functions introduced in class can be derived from the procedure used here, and essentially combine the spectra of the primaries and the response curve of the sensors into one curve.