Homework 5 - CURVES

Gleicher
Sample Solutions

1. $f(u) = au^3 + bu^2 + cu + d \qquad f'(u) = 3au^2 + 2bu + c$

$p_0 = f(0) = d$

$p_1 = f(.5) = a\left(\frac{1}{2}\right)^3 + b\left(\frac{1}{2}\right)^2 + c\left(\frac{1}{2}\right) + d$

$p_2 = f'(.5) = 3a\left(\frac{1}{2}\right)^2 + 2b\left(\frac{1}{2}\right) + c$

$p_3 = f(1) = a + b + c + d$

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1/8 & 1/4 & 1/2 & 0 \\ 3/4 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} -4 & 0 & -4 & 4 \\ 4 & -4 & 6 & -4 \\ -1 & 4 & -2 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

NOTE: FINDING M^{-1} would get full credit

HW 5

Gleicher
Sample

2. $f(u) = a_1 u^3 + b_1 u^2 + c_1 u + d_1$ ← first segment
$g(u) = a_2 u^3 + b_2 u^2 + c_2 u + d_2$ ← second segment

first segment:

$p0 = f(0) = d_1$
$p1 = f'(0) = c_1$
$p2 = f''(0) = 2b_1$
$p3 = f(1) = a_1 + b_1 + c_1 + d_1$

$$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \\ d_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -\frac{1}{2} & 1 \\ & & \frac{1}{2} & \\ & 1 & & \\ 1 & & & \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{bmatrix}$$

⇓

$a_1 = -p_0 - p_1 - \frac{1}{2} p_2 + p_3$
$b_1 = \frac{1}{2} p_2$
$c_1 = p_1$
$d_1 = p_0$

second segment:

$g(0) = p_3 = d_2$
$g'(0) = f'(1) = c_2$
$g''(0) = f''(1) = 2b_2$
$g(1) = p_4 = a_2 + b_2 + c_2 + d_2$

$f'(1) = 3a_1 u^2 + 2b_1 u + c_1 = 3a_1 + 2b_1 + c_1$
$f''(1) = 6a_1 u + 2b_1$

$d_2 = p_3$

$c_2 = 3a_1 + 2b_1 + c_1 = 3\left(-p_0 - p_1 - \frac{1}{2} p_2 + p_3\right) + 2\left(\frac{1}{2} p_2\right) + p_1$

$= -3p_0 - 2p_1 - \frac{1}{2} p_2 + 3p_3$

$b_2 = \frac{1}{2}\left(6a_1 + 2b_1\right) = 3\left(-p_0 - p_1 - \frac{1}{2} p_2 + p_3\right) + \frac{1}{2} p_2$

$= -3p_0 - 3p_1 - p_2 + 3p_3$

$a_2 = p_4 - b_2 - c_2 - d_2 = p_4 - (-3p_0 - 3p_1 - p_2 + 3p_3) - (-3p_0 - 2p_1 - \frac{1}{2} p_2 + 3p_3) - p_3$

$p_4 + 6p_0 + 5p_1 + \frac{3}{2} p_2 - 5p_3$

this checks out

this seems wrong

HW5

Check #2

Example Straight Line should work out

$p_0 = 0$ $\quad p_1 = 1$
$p_3 = 1$ $\quad p_2 = 0$
$p_4 = 2$

$f(u) = \;.\;.\; = a_1 = -0 - 1 - 0 + 1 = 0$
$b_1 = 0$
$c_1 = 1$
$d_1 = 0$

$= u$ (yes!)

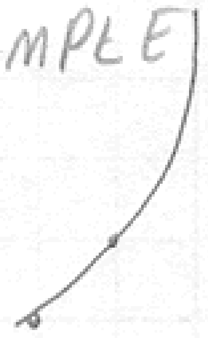
$g(u) = a_2 = 2 + 6\cdot 0 + 5\cdot 1 + \frac{3}{2}\cdot 0 - 5\cdot 1$

$b_2 = -3\cdot 0 - 3\cdot 1 - 0 + 3 = 0$

$c_2 = -3\cdot 0 - 2\cdot 1 - \frac{1}{2}\cdot 0 + 3\cdot 1 = 1$

$d_2 = 1$

EXAMPLE #2 Parabola



$x = t^2$
$x' = 2t$
$x'' = 2$

$y = (t+1)^2$
$y = t^2 + 2t + 1$

$p_0 = 0$ $\quad p_1 = 0$
$p_3 = 1$ $\quad p_2 = 2$
$p_4 = 4$

$a_1 = 0 - 0 - \frac{1}{2}2 + 1 = 0$
$b_1 = 1$
$c_1 = 0$
$d_1 = 0$

$a_2 = 4 + 6\cdot 0 + 5\cdot 0 + \frac{3}{2}\cdot 2 - 5\cdot 1$

$b_2 = -3\cdot 0 - 3\cdot 0 - 1\cdot 2 + 3 = 1$

$c_2 = -3\cdot 0 - 2\cdot 0 - \frac{1}{2}\cancel{2} + 3\cdot 1 = 2$

$d_2 = 1$

HW 5

#3

$$f = au^5 + bu^4 + cu^3 + du^2 + eu + f$$
$$f' = 5au^4 + 4bu^3 + 3cu^2 + 2du + e$$

$$\begin{aligned}
p_0 &= f(0) &&= f \\
p_1 &= f'(0) &&= e \\
p_2 &= f(.5) &&= \left(\tfrac{1}{2}\right)^5 a + \left(\tfrac{1}{2}\right)^4 b + \left(\tfrac{1}{2}\right)^3 c + \left(\tfrac{1}{2}\right)^2 d + \tfrac{1}{2}e + f \\
p_3 &= f'(.5) &&= 5\left(\tfrac{1}{2}\right)^4 a + 4\left(\tfrac{1}{2}\right)^3 b + 3\left(\tfrac{1}{2}\right)^2 c + 2\left(\tfrac{1}{2}\right) d + e \\
p_4 &= f(1) &&= a + b + c + d + e + f \\
p_5 &= f'(1) &&= 5a + 4b + 3c + 2d + e
\end{aligned}$$

$$\begin{matrix} a \\ b \\ c \\ d \\ e \\ f \end{matrix} = \begin{bmatrix} & & & & & 1 \\ & & & & 1 & \\ 1/32 & 1/16 & 1/8 & 1/4 & 1/2 & 1 \\ 5/16 & 4/8 & 3/4 & 2/2 & 1 & \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 5 & 4 & 3 & 2 & 1 & \end{bmatrix}^{-1} \begin{bmatrix} p \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 4 & 0 & 16 & -24 & 4 \\ -68 & -12 & 16 & -40 & 52 & -8 \\ 66 & 13 & -32 & 32 & -34 & 5 \\ -23 & -6 & 16 & -8 & 7 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

HW 5 #4

* The QUINTIC IS C^2, the HERMITES ARE NOT
* The Hermite HAS LOCAL CONTROL (p_0, p_1 does not affect the second curve segment), the QUINTIC DOES NOT
* The QUINTIC MAY HAVE EXTRA WIGGLES