CS 638
Homework 1: Linear Algebra and Geometry Review

Due: Thursday, September 9, 1999 9:30AM (the beginning of class)

Please turn in your answers with your completed academic conduct form and course survey.

Notations:

Scalars are denoted as non-bold, lower case letters (x,y,z)
Vectors are denoted as bold, lower case letters (**x**, **y**, **z**)
Matrices are denoted as bold, upper case letters (**X**, **Y**, **Z**)
The dot product of vectors **x** and **y** is denoted **x•y**
The vector cross product of **x** and **y** is denoted **x**×**y**
The absolute value of a scalar is denoted by |x|

Question 1:

Let **A** and **B** be 4 by 4 matrices, and **c** be a length 4 column vector.

Note that to compute **ABc** it is faster to first multiply **Bc**.

Suppose, however, that we have a large number of 4-vectors and we want to multiply each on by **AB**. For a large enough number of vectors, it will be faster to compute **AB** and then multiply this intermediate result by each vector. How many vectors do we need to process in this way for it to be more efficient to compute this way?

Question 2:

Consider the plane defined in 3 space 2x+2y+z=2.

What point on this plane is closest to the origin?

Does this plane intersect a unit sphere centered at the original?

Question 3:

Is the space spanned by the following vectors a line, plane, or something larger?

3A: **a**=(1,2,3), **b**=(2,4,6), **c**=(3,6,9)

3B: **a**=(1,2,3), **b**=(-2,-4,-6), **c**=(3,6,9)

3C: **a**=(1,2,3), **b**=(2,-4,6), **c**=(3,6,9)

3D: **a**=(4,0,3,-2), **b**==(-8,0,-6,4), **c**=(-2,0,-3/2,1)

Question 4:

a and **b** are unit 3-vectors , Let **c**=**a**×**b**, and **d**=**a**×**c**

If the vectors **a, c,** and **d** do not form a basis for 3-space, what values can **a•b** have?

Question 5:

Prove that if there are 3 points on the plane (x1, y1), (x2,y2), (x3,y3), that the determinant of the 3 by 3 Matrix $\begin{matrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{matrix}$ has determinant 0 if the three points are co-linear.