Modified Floyd-Warshall Algorithm for Risk Arbitrage

Ch. Mouratidis
TEI of Western Macedonia
Department of Business and Administration
50 100, Koila Kozanis
Greece
cmourati@teikoz.gr

G. Majchrowska
TEI of Western Macedonia
Department of Information Technology
Applications in Administration and Economy
51 100, Grevena
Greece
grazyna@csd.auth.gr

D. Zissopoulos
TEI of Western Macedonia
Department of Business and Administration
50 100, Koila Kozanis
Greece
dr@teikoz.gr

N. Asimopoulos
TEI of Western Macedonia
Department of Electrical Engineering
50 100, Koila Kozanis
Greece
asimopou@teikoz.gr

Abstract: Traditionally risk arbitrage is the simultaneous purchase of stock in a company being acquired and the sale of stock of the acquirer. Modern risk arbitrage focuses on capturing the spreads between the market value of an announced takeover target and the eventual price at which the acquirer will buy the target's shares. Here we look at the concept of arbitrage, how market makers utilize "true arbitrage," and, finally, how retail investors can take advantage of arbitrage opportunities.

Key–Words: Arbitrage, Risk Arbitrage, Weighted Graphs.

1 Concepts of Arbitrage

Arbitrage, in its purest form, is defined as the purchase of securities on one market for immediate resale on another market in order to profit from a price discrepancy, see [2], [3], [4]. This results in immediate risk-free profit. For example, if a security's price on the NYSE, New York Stock Exchange, is trading out of sync with its corresponding futures contract on Chicago's exchange, a trader could simultaneously sell (short) the more expensive of the two and buy the other, thus profiting on the difference, see [5]. This type of arbitrage requires the violation of at least one of the following three conditions:

- The same security must trade at the same price on all markets.

- Two securities with identical cash flows must trade at the same price.

- A security with a known price in the future (via

a futures contract) must trade today at that price discounted by the risk-free rate.

Arbitrage, however, can take other forms. Risk arbitrage (or statistical arbitrage) is the second form of arbitrage that we will discuss. Unlike pure arbitrage, risk arbitrage entails–you guessed it–risk. Although considered "speculation," risk arbitrage has become one of the most popular (and retail-trader friendly) forms of arbitrage.

Here's how it works: let's say Firm A may make an offer to acquire Firm B by exchanging one share of its own stock for two shares of Firm B's stock. If the stock of Firm A is trading at $50 and the stock of Firm B is trading at $23, the risk arbitrager would buy shares in Firm B and sell short one-half this number of shares in Firm A. If the buyout offer is approved, the two stocks will exchange on a one-for-two basis and the arbitrage position will be profitable. The risk is that the buyout will be unsuccessful and the exchange of stock will not take place.

2 Retail Traders: Risk Arbitrage

Despite the disadvantages in pure arbitrage, risk arbitrage is still accessible to most retail traders. Although this type of arbitrage requires taking on some risk, it is generally considered "playing the odds." Here we will examine some of the most common forms of arbitrage available to retail traders.

Market makers have several advantages over retail traders:

- Far more trading capital

- Generally more skill

- Up-to-the-second news

- Faster computers

- More complex software

- Access to the dealing desk

Combined, these factors make it nearly impossible for a retail trader to take advantage of pure arbitrage opportunities. Market makers use complex software that is run on top-of-the-line computers to locate such opportunities constantly. Once found, the differential is typically negligible, and requires a vast amount of capital in order to profit–retail traders would likely get burned by commission costs. Needless to say, it is almost impossible for retail traders to compete in the risk-free genre of arbitrage.

A merger arbitrageur looks at the risk that the merger deal will not close on time, or at all. Because of this slight uncertainty, the target company's stock will typically sell at a discount to the price that the combined company will have when the merger is closed.

A regular portfolio manager may focus only on the profitability of the merged entity. In contrast, merger arbitrageurs care only about the probability of the deal being approved and how long it will take the deal to close.

The example of risk arbitrage we saw above demonstrates takeover and merger arbitrage, and it is probably the most common type of arbitrage. It typically involves locating an undervalued company that has been targeted by another company for a takeover bid. This bid would bring the company to its true, or intrinsic, value. If the merger goes through successfully, all those who took advantage of the opportunity will profit handsomely; however, if the merger falls through, the price may drop.

The key to success in this type of arbitrage is speed; traders who utilize this method usually trade on Level II and have access to streaming market news. The second something is announced, they try to get in on the action before anyone else.

3 Risk Evaluation

Risk evaluation is concerned with assessing probability and impact of individual risks, taking into account any interdependencies or other factors outside the immediate scope under investigation: Probability is the evaluated likelihood of a particular outcome actually happening (including a consideration of the frequency with which the outcome may arise). For example, major damage to a building is relatively unlikely to happen, but would have enormous impact on business continuity. Conversely, occasional personal computer system failure is fairly likely to happen, but would not usually have a major impact on the business. Impact is the evaluated effect or result of a particular outcome actually happening.

Impact should ideally be considered under the elements of:

- time

- quality

- benefit

- people/resource

Some risks, such as financial risk, can be evaluated in numerical terms. Others, such as adverse publicity, can only be evaluated in subjective ways. There is a need for some framework for categorising risks, for example, high, medium and low. When considering a risk's probability, another aspect is when the risk might occur. Some risks will be predicted to be further away in time than others and so attention can be focused on the more immediate ones. This prediction is called the risk's proximity. The proximity of each risk should be included in the Risk Log.

Let's say you aren't among the first in, however. How do you know if it is still a good deal? Well, one way is to use Benjamin Graham's risk-arbitrage formula to determine optimal risk/reward. His equations state the following:

$$AnnualReturn = \frac{C * G - L * (100\% - C)}{Y * P}$$

Where:

- C is the expected chance of success (%).

- P is the current price of the security.

- L is the expected loss in the event of a failure (usually original price).

- Y is the expected holding time in years (usually the time until the merger takes place).

- G is the expected gain in the event of a success (usually takeover price).

For example, if $C = 70\%$, $G = 0.15$, $L = 0.20$, $P = 1$ and $Y = 6$ months, then

$$Annual Return =$$

$$(0.70 * 0.15 - 0.20 * (1 - 0.70)) / (0.5 * 1) = 0.09 = 9\%$$

Granted, this is highly empirical, but it will give you an idea of what to expect before you get into a merger arbitrage situation.

Graham's formula can be used to evaluate the potential return on the risk arbitrage operation in the Acme and Smith merger. The expected gain in the event of success is $1.00 (the spread between the $24.00 quoted price on the open market and the $25 Acme tender offer). If the merger fails to occur, the Smith stock may fall to its pre-tender offer of $15 per share (in many cases, history has proven otherwise; once a company is "in play" as a takeover target, its stock may remain inflated in anticipation of another acquirer materializing. We shall disregard this possibility for the sake of conservatism). Hence, the expected loss in points in the event of failure is $9. Assume there are no antitrust concerns, so the likelihood of consummation is 95%. Also assume the investor expects to hold his shares for one month (1/12 or 8.33% of a year) until the transaction is complete. The current price of the security is $15 per share. Plugging these into Graham's formula, the investor gets the following:

$$Indicated\ annual\ return =$$

$$[1 * .95 - 9(1.00 - .95)] \div .0833 * 24 = 25\%$$

In other words, had the investor been able to earn the same return on his capital for the entire year as he did during the holding period of this investment, he would have earned twenty five percent. In a world where the historic annual return on long-term equities has hovered around twelve percent, this is mouth watering.

4 Liquidation Arbitrage

This is the type of arbitrage Gordon Gekko employed when he bought and sold off companies. Liquidation arbitrage involves estimating the value of the company's liquidation assets. For example, say Company A has a book (liquidation) value of $10/share and is currently trading at $7/share. If the company decides to liquidate, it presents an opportunity for arbitrage. In Gekko's case, he took over companies that he felt would provide a profit if he broke them apart and sold them–a practice employed in reality by larger institutions.

A version of Benjamin Graham's risk arbitrage formula used for takeover and merger arbitrage can be employed here. Simply replace the takeover price with the liquidation price, and holding time with the amount of time before liquidation.

Pairs trading (also known as relative-value arbitrage) is far less common than the two forms discussed above. This form of arbitrage relies on a strong correlation between two related or unrelated securities. It is primarily used during sideways markets as a way to profit.

Here's how it works. First, you must find "pairs." Typically, high-probability pairs are big stocks in the same industry with similar long-term trading histories. Look for a high percent correlation. Then, you wait for a divergence in the pairs between 5-7% divergence that lasts for an extended period of time (2-3 days). Finally, you can go long and/or short on the two securities based on the comparison of their pricing. Then, just wait until the prices come back together.

One example of securities that would be used in a pairs trade is GM and Ford. These two companies have a 94% correlation. You can simply plot these two securities and wait for a significant divergence; then chances are these two prices will eventually return to a higher correlation, offering opportunity in which profit can be attained. (For more on this subject, see "Finding Profit in Pairs.")

5 Find Opportunity

Many of you may be wondering where you can find these accessible arbitrage opportunities. The fact is much of the information can be attained with tools that are available to everyone. Brokers typically provide newswire services that allow you to view news the second it comes out.

Level II is essentially the order book for Nasdaq stocks. When orders are placed, they are placed through many different market makers and other market participants. Level II will show you a ranked list of the best bid and ask prices from each of these participants, giving you detailed insight into the price action. Knowing exactly who has an interest in a stock can be extremely useful, especially if you are day trading.

Level II trading is also an option for individual traders and can give you an edge. Finally, screening software can help you locate undervalued securities (that have appropriate price/book ratio, PEG ratio, etc.).

Arbitrage opportunities can be located also in the following way.

Let us consider a weighted and directed graph G with $V = \{v_1, v_2, ..., v_N\}$ being a set of vertices and E a set of edges. In our case the vertices correspond to the shares and the edges to the exchanging rates between them in different currencies i.e. edges in a graph are labelled with a weight (exchange rate). Such a graph is referred to as a weighted graph. A weighted graph is generally implemented using an adjacency matrix: we use an $N \times N$ array of integers or reals, where the number indicates the weight of an edge.

Here the adjacency matrix is the exchange table of the shares, i.e. the value $a_{i,j}$ in the matrix is the exchange rate from share u_i to the share u_j. In the main diagonal of the matrix we have the values $a_{i,i} = 1$ for all $i = 1, 2, ..., N$. So, to have profitable arbitrage we need to find a cycle in the graph

$$u_{i_0}, u_{i_1}, u_{i_2}, ..., u_{i_n} = u_{i_1} \ for \ some \ n \leq N \quad (1)$$

such that the value

$$P = \prod_{j=0}^{n-1} a_{i_j, i_{j+1}} = a_{i_0, i_1} a_{i_1, i_2} ... a_{i_{n-1}, i_0} > 1.$$

Then the value $P - 1$ will be our pure profit from the arbitrage between the shares of the cycle 1. This problem falls in a general category of finding path between two nodes in a graph with minimum (or due to duality properties with maximum) weight.

Since in the graph we do not use negative weights, the most efficient algorithm for this kind of problems in Floyd-Warshall algorithm.

It compares all possible paths through the graph between each pair of vertices. We consider a 3-dimentional matrix $[sp_{i,j}^k]$ for $i, j, k = 1, 2, ..., N$, where $sp_{i,j}^k$ is the shortest possible path from i to j using only vertices 1 through k as intermediate points along the way.

The idea of computing $sp_{i,j}^k$, as far as we know the values $sp_{i,j}^{k-1}$ is the following:

for the shortest path from i to j using only nodes 1 to k there are two candidates. Either the true shortest path uses only the nodes from 1 to $k-1$ or there exists some path that goes from i to k and then from k to j that is better. Therefore, we can define $sp_{i,j}^k$ in terms of the follownig recursive formula:

$$sp_{i,j}^k := \min(sp_{i,j}^{k-1}, sp_{i,k}^{k-1} + sp_{k,j}^{k-1}) \ \ k = 1, ...n$$

with $sp_{i,j}^0 = a_{i,j}$.

This formula is the heart of the Floyd-Warshall algorithm. It works by computing $sp_{i,j}^1$ for all (i, j) pairs, then using that to find $sp_{i,j}^2$ etc. This continues until $k = N$. Floyd-Warshall algorithm is given below.

Input: Graph adjustency matrix
Output: All-pairs shortest path
for $k = 1$ *to* N **do**

    for $i = 1$ *to* N **do**

        for $j = 1$ *to* N **do**

            $sp_{i,j}^k = \min(sp_{i,j}^{k-1}, sp_{i,k}^{k-1} + sp_{k,j}^{k-1})$

        end

    end

end

    Algorithm 1: Floyd-Warshall Algorithm

Using this algorithm we solve the shortest path problem in time proportional to N^3.

To solve arbitrage problems, we have to do major modifications in the previous algorithm, because of two reasons. First of all, we want the knowledge of the profit of arbitrage to have also the way to achieve this. So, we shall also need a second matrix to hold the path we have to follow from the share u_i to the share u_j in order to have profit. This is not a real computational problem, since we can do it in $O(N^3)$ time of Floyd-Warshall algorithm.

The computational problem arrives from the nature of our graph. As we already said if an arbitrage opportunity exists, then market forces should eliminate it very quickly. So we are more interested in short paths than in the long ones. That means, we prefer to do less exchanges with smaller profit than more exchanges with bigger profit, because of the risk that in the mean time the arbitrage opportunity will be eliminated.

We need one more variable in the main matrix to hold the length of the path, to pass from the share u_i to the share u_j. We consider the 3-dimensional matrix $[rt_{i,j}^s]$ for $i, j, s = 1, 2, ..., N$ which will be the best profit one can have, going from i to j in s steps. Unfortunatelly, this option will increase the computational time, it means we can do it in time proportional to N^4. But, since we are interrested in values of N smaller than 30, we need to add to the computational time just a fraction of a second.

We will use also another 3-dimensional matrix $p_{i,j}^s$. Each element $p_{i,j}^s$ describes the last step (last intermediate vertice) from the path going from i to j in s steps which corresponds to the $rt_{i,j}^s$. The initializations of the two matrices we are going to use are as follows:

$$rt_{i,j}^s = 0, \quad i, j, s = 1, 2, ..., N$$
$$rt_{i,i}^1 = 1, \quad i = 1, 2, ..., N$$
$$p_{i,j}^1 = 0, \quad i, j = 1, 2, ..., N$$

Input: Shares exchange table
Output: Profit between shares
for $s = 2$ *to* N **do**
    for $k = 1$ *to* N **do**
        for $i = 1$ *to* N **do**
            for $j = 1$ *to* N **do**
                $hld = rt_{i,k}^{s-1} * rt_{k,j}^1$
                if $hld > rt_{i,j}^s$ **then**
                    $rt_{i,j}^s := hld$;
                    $path_{i,j}^s := k$;
                end
            end
        end
    end
end

Algorithm 2: Arbitrage algorithm

When the algorithm finishes the matrix $rate$ consists of the values of profit one can get going from i to j in s steps. It means that $rate_{i,i}^s$ gives as the maximal gain we can obtain starting and ending with the share i. If $rate_{i,i}^s > 1$ for some s, then we can do arbitrage. And the exchanges we have to do to get the gain are given in the matrix $path$. If the path from the vertice i to the vertice j in s steps is $i = i_1, i_2, i_3, ..., i_{s-1}, i_s = j$ then the (i, j, s)-element of the matrix $path$, this means $path_{i,j}^s$ is equal to i_{s-1}, $path_{i,i_{s-1}}^{s-1} = i_{s-2}$, $path_{i,i_{s-2}}^{s-2} = i_{s-3}$ etc. In this way we can obtain the path of all the exchanges we have to do to gain from arbitrage. Using this algorithm we can find all arbitrage possibilities in time proportional to N^4.

As an example, we used the above algorithm for $N = 20$ on an Intel Centrino $1.8GHz$ processor. We got the result in 0.7 seconds.

6 Conclusion

Arbitrage is a very broad form of trading that encompasses many strategies; however, they all seek to take advantage of increased chances of success. Although the risk-free forms of pure arbitrage are typically unavailable to retail traders, there are several high-probability forms of risk arbitrage that offer retail traders many opportunities to profit.

References:

[1] Bjork T., (2004). *Arbitrage Theory in Continuous Time*. Oxford University Press.

[2] Jiang Wang (2003). *Arbitrage Pricing Theory*. Lecture Notes, The MIT Press.

[3] Bansal Ravi, S. Viswanathan (1993). *No Arbitrage and Arbitrage Pricing: A New Approach*. Journal of Finance: 1231-1262.

[4] Priestley Richard (1996). *The Arbitrage Pricing Theory, Macroeconomic and Financial Factors, and Expectations Generating Processes*. Journal of Banking and Finance 20: 869-890.

[5] Mouratidis Ch., Majchrowska G., Charitoudi G. (2009). *Weighted Graphs Models in Searching for Arbitrage Oppotrunities* accepted to Mathematical Modelling and Applied Computing.

[6] T. H. Cormen, C. E. Leiserson, R. L. Rivest, C. Stein (2001). *Introduction to Algorithms*. The MIT Press.

[7] W. Kocay, D. L. Kreher (2005) *Graphs, Algorithms and Optimization*. Chapman & Hall/CRC Press.