Online Knowledge-Based Support Vector Machines

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Outline

- ***Knowledge-Based Support Vector Machines***
- The Adviceptron: Online KBSVMs
- A Real-World Task: Diabetes Diagnosis
- A Real-World Task: Tuberculosis Isolate Classification
- Conclusions

Knowledge-Based SVMs

- Introduced by Fung et al (2003)
- Allows incorporation of expert advice into SVM formulations
- Advice is specified with respect to polyhedral regions in input (feature) space

$$(\texttt{feature}_7 \geq 5) \wedge (\texttt{feature}_{12} \leq 4) \Rightarrow (\texttt{class} = +1)$$
$$(\texttt{feature}_2 \leq -3) \wedge (\texttt{feature}_3 \leq 4) \wedge (\texttt{feature}_{10} \geq 0) \Rightarrow (\texttt{class} = -1)$$
$$(3\texttt{feature}_6 + 5\texttt{feature}_8 \geq 2) \wedge (\texttt{feature}_{11} \leq -3) \Rightarrow (\texttt{class} = +1)$$

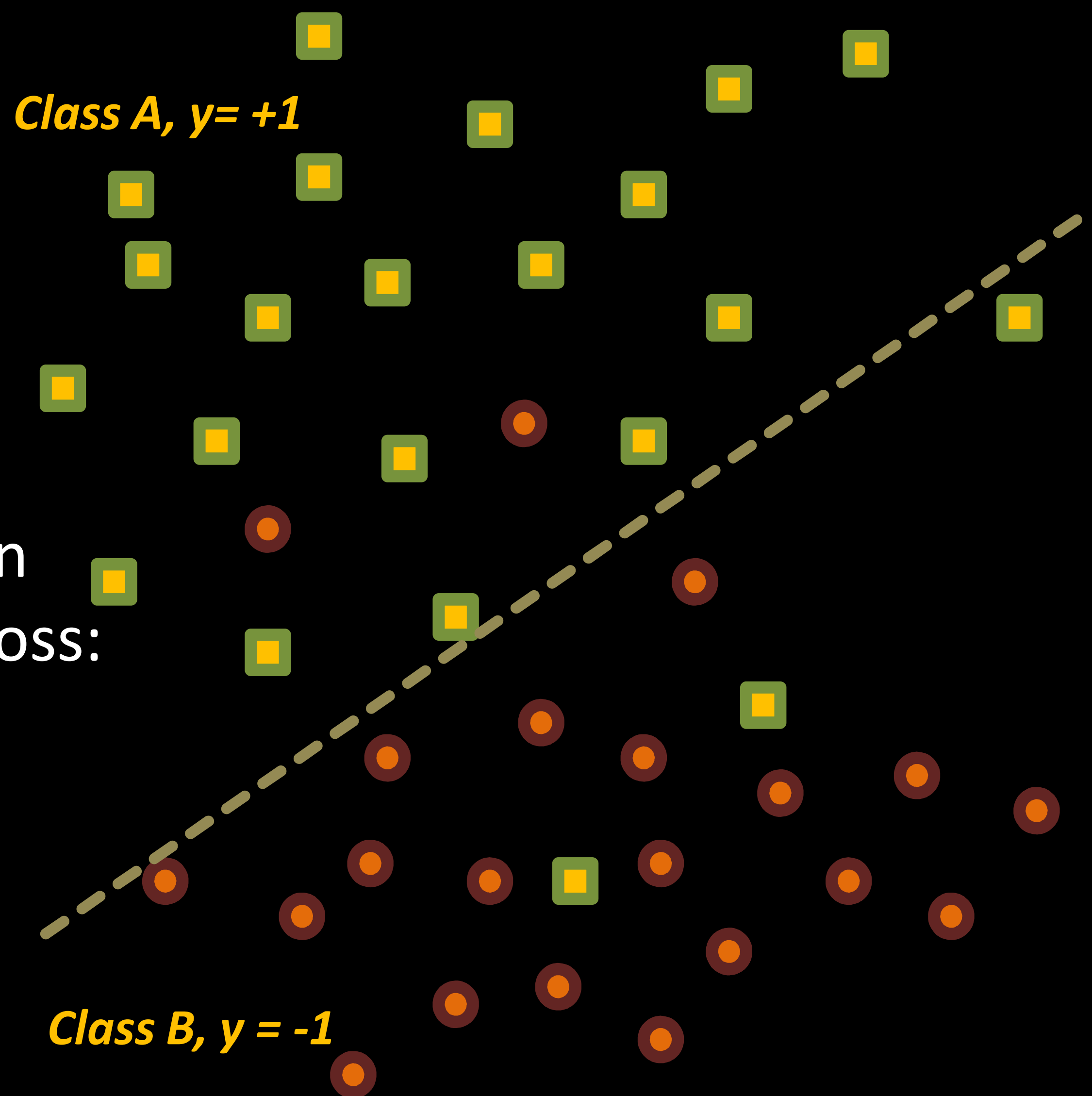
- Can be incorporated into SVM formulation as constraints using ***advice variables***

Knowledge-Based SVMs

In classic SVMs, we have *T* labeled data points $(\mathbf{x}^t, y_t)$, $t = 1, \ldots, T$. We learn a linear classifier $\mathbf{w}'\mathbf{x} - b = 0$.

The standard SVM formulation trades off regularization and loss:

$$\begin{aligned} \min \quad & \frac{1}{2}\|\mathbf{w}\|^2 + \lambda\,\mathbf{e}'\xi \\ \text{sub. to} \quad & Y(X\mathbf{w} - b\mathbf{e}) + \xi \geq \mathbf{e}, \\ & \xi \geq 0. \end{aligned}$$



Knowledge-Based SVMs

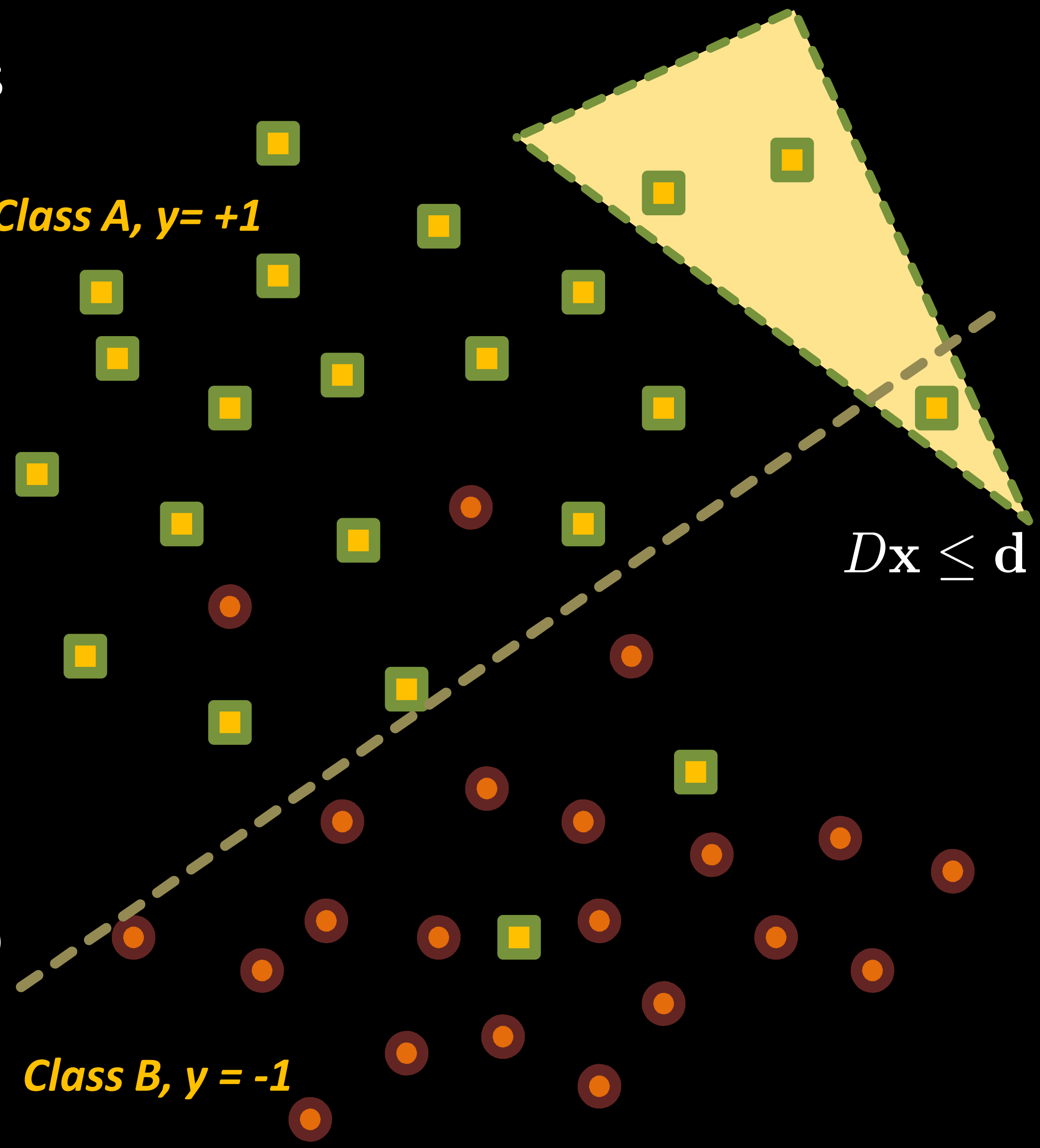
We assume an expert provides polyhedral advice of the form

$$D\mathbf{x} \leq \mathbf{d} \Rightarrow \mathbf{w}'\mathbf{x} \geq b$$

We can transform the logic constraint above using ***advice variables, u***

$$\begin{aligned} D'\mathbf{u} + \mathbf{w} &= 0, \\ -\mathbf{d}'\mathbf{u} - b &\geq 0, \\ \mathbf{u} &\geq 0 \end{aligned}$$

These constraints are added to the standard formulation to give ***Knowledge-Based SVMs***



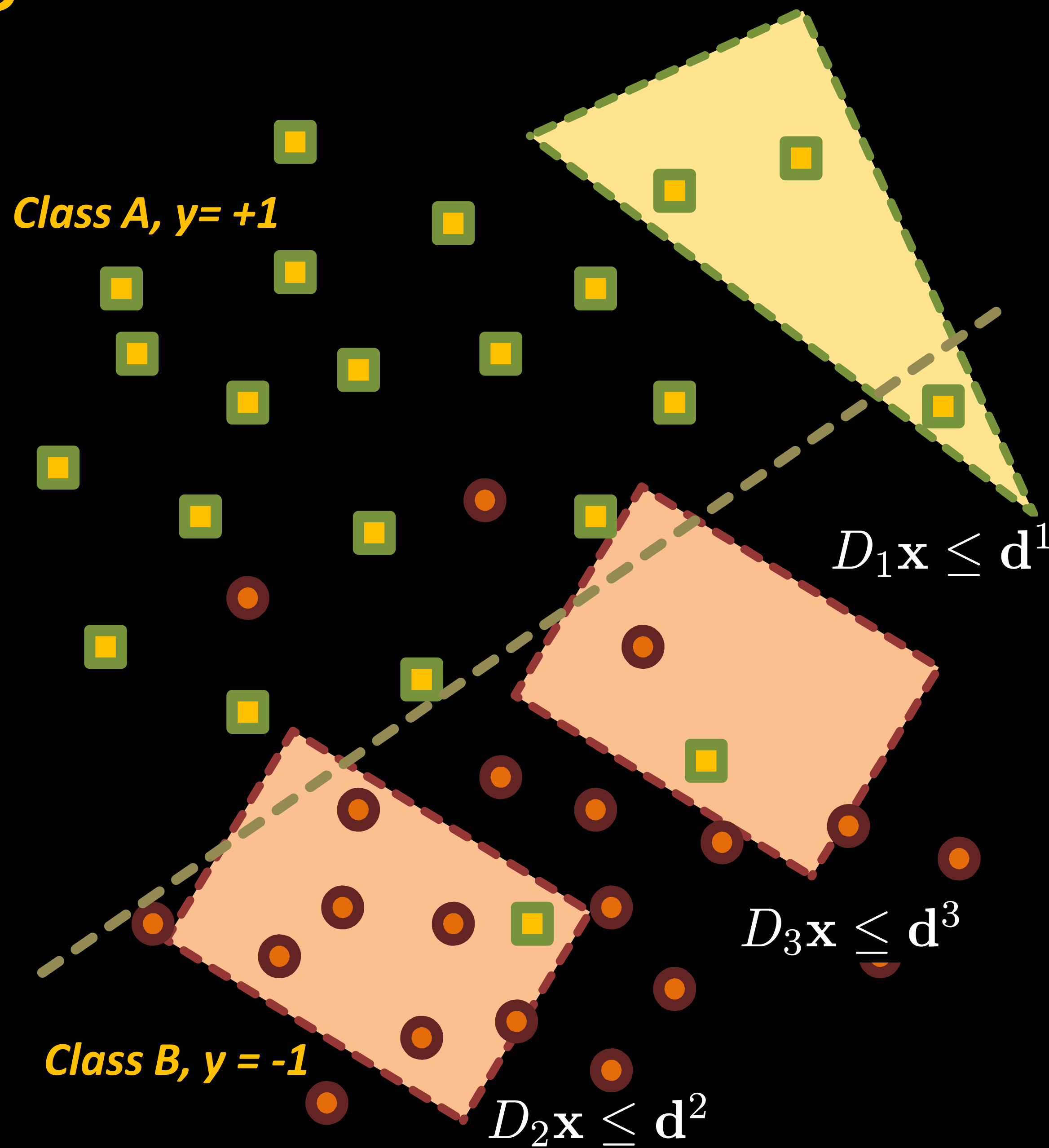
Knowledge-Based SVMs

In general, there are ***m* advice sets**, each with label $z = \pm 1$ for advice belonging to Class A or B,

$$D_i \mathbf{x} \leq \mathbf{d}^i \Rightarrow z_i\left(\mathbf{w}'\mathbf{x}\right) - b \geq 0$$

Each advice set ***adds the following constraints*** to the SVM formulation

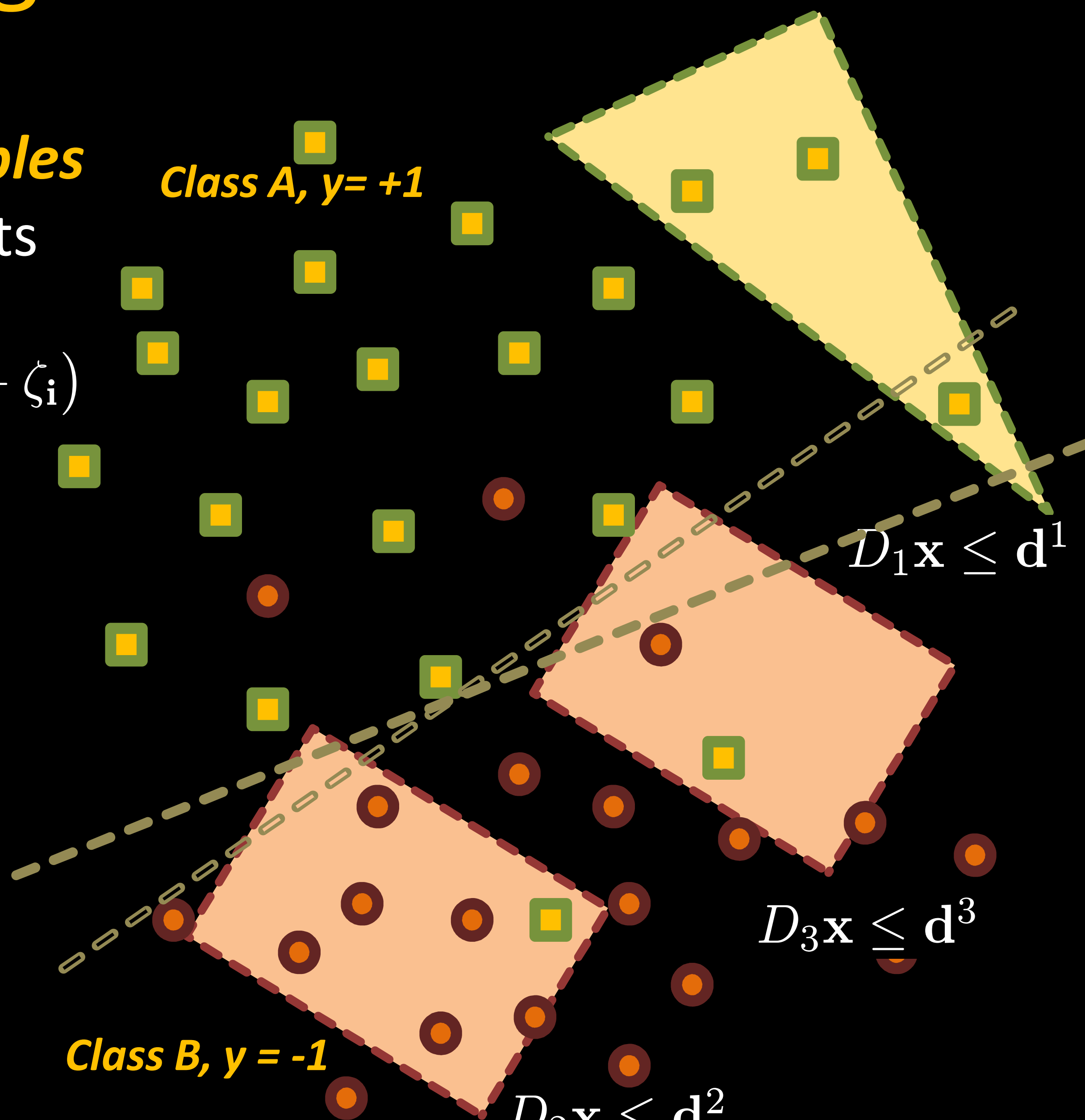
$$\begin{aligned} D_i' \mathbf{u}^i + z_i \mathbf{w} &= 0, \\ -\mathbf{d}^{i\prime} \mathbf{u}^i - z_i b &\geq 0, \\ \mathbf{u}^i &\geq 0 \end{aligned}$$



Knowledge-Based SVMs

The batch KBSVM formulation introduces ***advice slack variables*** to ***soften*** the advice constraints

$$
\begin{aligned}
\min \quad & \frac{1}{2}\|\mathbf{w}\|^{\mathbf{2}} + \lambda\,\mathbf{e}'\xi + \mu \sum_{\mathbf{i=1}}^{\mathbf{m}} \left(\eta^{\mathbf{i}} + \zeta_{\mathbf{i}}\right) \\
\text{s.t.} \quad & Y(X\mathbf{w} - b\mathbf{e}) + \xi \geq \mathbf{e}, \\
& \xi \geq 0, \\
& D_i'\mathbf{u}^i + z_i\mathbf{w} + \eta^i = 0, \\
& -\mathbf{d}^{i\prime}\mathbf{u}^i - z_i b + \zeta_i \geq 1, \\
& \mathbf{u}^i, \eta^i, \zeta_i \geq 0,\ i = 1, \ldots, m.
\end{aligned}
$$



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Online KBSVMs

- Need to derive an ***online version of KBSVMs***
- Algorithm is provided with advice and ***one*** labeled data point at each round
- Algorithm should ***update the hypothesis*** at each step, $\boldsymbol{w}^t$, ***as well as*** *the* ***advice vectors***, $\boldsymbol{u}^{i,t}$

Passive-Aggressive Algorithms

- Adopt the framework of ***passive-aggressive algorithms*** (Crammer et al, 2006), where at each round, when a new data point is given,
  - if loss = 0, there is no update (**passive**)
  - if loss > 0, update weights to minimize loss (**aggressive**)
- Why passive-aggressive algorithms?
  - readily applicable to **most SVM losses**
  - possible to derive elegant, **closed-form update rules**
  - simple rules provide fast updates; **scalable**
  - analyze performance by deriving **regret bounds**

Online KBSVMs

- There are *m* advice sets, $(D_i,\, \mathbf{d}^i,\, z_i)_{i=1}^m$
- At round *t*, the algorithm receives $(\mathbf{x}^t,\, y_t)$
- The current hypothesis is $\mathbf{w}^t$, and the current advice variables are $\mathbf{u}^{i,t},\, i = 1, \ldots, m$

At round *t*, the formulation for deriving an update is

$$\begin{array}{cl}
\min\limits_{\xi, \mathbf{u}^i, \eta^i, \zeta_i \geq 0, \mathbf{w}} & \frac{1}{2}\|\mathbf{w} - \mathbf{w}^t\|^2 + \frac{1}{2}\sum_{i=1}^m \|\mathbf{u}^i - \mathbf{u}^{i,t}\|^2 + \frac{\lambda}{2}\xi^2 + \frac{\mu}{2}\sum_{i=1}^m \left(\|\eta^i\|^2 + \zeta_i^2\right) \\
\text{subject to} & y_t \mathbf{w}'\mathbf{x}^t - 1 + \xi \geq 0, \\
& \left.\begin{array}{r} D_i'\mathbf{u}^i + z_i \mathbf{w} + \eta^i = 0 \\ -\mathbf{d}^{i'}\mathbf{u}^i - 1 + \zeta_i \geq 0 \\ \mathbf{u}^i \geq 0 \end{array}\right\} \; i = 1, \ldots, m.
\end{array}$$

Formulation At The *t*-th Round

- There are *m* advice sets, $(D_i,\ \mathbf{d}^i,\ z_i)_{i=1}^m$
- At round *t*, the algorithm receives $(\mathbf{x}^t,\ y_t)$
- The current hypothesis is $\mathbf{w}^t$, and the current advice variables are $\mathbf{u}^{i,t},\ i = 1, \ldots, m$

proximal terms for hypothesis and advice vectors

$$
\begin{array}{cl}
\displaystyle\min_{\xi, \mathbf{u}^i, \eta^i, \zeta_i \geq 0, \mathbf{w}} & \displaystyle \frac{1}{2}\|\mathbf{w} - \mathbf{w}^t\|^2 + \frac{1}{2}\sum_{i=1}^m \|\mathbf{u}^i - \mathbf{u}^{i,t}\|^2 + \frac{\lambda}{2}\xi^2 + \frac{\mu}{2}\sum_{i=1}^m \left(\|\eta^i\|^2 + \zeta_i^2\right), \\
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$$
\begin{array}{rl}
\min\limits_{\xi, \mathbf{u}^i, \eta^i, \zeta_i \geq 0, \mathbf{w}} & \frac{1}{2}\|\mathbf{w} - \mathbf{w}^t\|^2 + \frac{1}{2}\sum_{i=1}^m \|\mathbf{u}^i - \mathbf{u}^{i,t}\|^2 + \frac{\lambda}{2}\overset{\text{data loss}}{\xi^2} + \frac{\mu}{2}\sum_{i=1}^m \overset{\text{advice loss}}{\left(\|\eta^i\|^2 + {\zeta_i}^2\right)}, \\
\text{subject to} & y_t \mathbf{w}'\mathbf{x}^t - 1 + \xi \geq 0, \\
& \left.\begin{array}{r} D_i'\mathbf{u}^i + z_i\mathbf{w} + \eta^i = 0 \\ -\mathbf{d}^{i\prime}\mathbf{u}^i - 1 + \zeta_i \geq 0 \\ \mathbf{u}^i \geq 0 \end{array}\right\} \; i = 1, ..., m.
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parameters

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\begin{aligned}
\min_{\xi, \mathbf{u}^i, \eta^i, \zeta_i \geq 0, \mathbf{w}} \quad & \frac{1}{2}\|\mathbf{w} - \mathbf{w}^t\|^2 + \frac{1}{2}\sum_{i=1}^m \|\mathbf{u}^i - \mathbf{u}^{i,t}\|^2 + \frac{\lambda}{2}\xi^2 + \frac{\mu}{2}\sum_{i=1}^m \left(\|\eta^i\|^2 + \zeta_i^2\right), \\
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\begin{array}{rl}
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\text{subject to} & y_t \mathbf{w}'\mathbf{x}^t - 1 + \xi \geq 0, \\
& \left.\begin{array}{r} D_i'\mathbf{u}^i + z_i \mathbf{w} + \eta^i = 0 \\ -\mathbf{d}^{i\prime}\mathbf{u}^i - 1 + \zeta_i \geq 0 \\ \mathbf{u}^i \geq 0 \end{array}\right\} i = 1, \ldots, m.
\end{array}
$$

inequality constraints make deriving a closed-form update impossible

Formulation At The *t*-th Round

- There are *m* advice sets, $(D_i, \mathbf{d}^i, z_i)_{i=1}^m$
- At round *t*, the algorithm receives $(\mathbf{x}^t, y_t)$
- The current hypothesis is $\mathbf{w}^t$, and the current advice-vector estimates are $\mathbf{u}^{i,t}$, $i = 1, \ldots, m$

DECOMPOSE INTO SMALLER SUBPROBLEMS

$$
\begin{aligned}
\min_{\xi, \mathbf{u}^i, \eta^i, \zeta_i \geq 0, \mathbf{w}} \quad & \frac{1}{2}\|\mathbf{w} - \mathbf{w}^t\|^2 + \frac{1}{2}\sum_{i=1}^{m}\|\mathbf{u}^i - \mathbf{u}^{i,t}\|^2 + \frac{\lambda}{2}\xi^2 + \frac{\mu}{2}\sum_{i=1}^{m}\left(\|\eta^i\|^2 + \zeta_i^2\right), \\
\text{subject to} \quad & y_t \mathbf{w}'\mathbf{x}^t - 1 + \xi \geq 0, \\
& \left.\begin{aligned} D_i'\mathbf{u}^i + z_i\mathbf{w} + \eta^i &= 0 \\ -\mathbf{d}^{i\prime}\mathbf{u}^i - 1 + \zeta_i &\geq 0 \\ \mathbf{u}^i &\geq 0 \end{aligned}\right\} \; i = 1, \ldots, m.
\end{aligned}
$$

inequality constraints make deriving a closed-form update impossible

Decompose Into *m+1* Sub-problems

- First sub-problem: update hypothesis by fixing the advice variables, to their values at the *t*-th iteration $\mathbf{u}^i = \mathbf{u}^{i,t}$

$$
\begin{aligned}
\min_{\xi, \mathbf{u}^i, \eta^i, \zeta_i \geq 0, \mathbf{w}} \quad & \frac{1}{2}\|\mathbf{w} - \mathbf{w}^t\|^2 + \frac{1}{2}\sum_{i=1}^{m}\|\mathbf{u}^i - \mathbf{u}^{i,t}\|^2 + \frac{\lambda}{2}\xi^2 + \frac{\mu}{2}\sum_{i=1}^{m}\left(\|\eta^i\|^2 + \zeta_i^2\right), \\
\text{subject to} \quad & y_t \mathbf{w}'\mathbf{x}^t - 1 + \xi \geq 0, \\
& \left.\begin{array}{r} D_i'\mathbf{u}^i + z_i\mathbf{w} + \eta^i = 0 \\ -\mathbf{d}^{i'}\mathbf{u}^i - 1 + \zeta_i \geq 0 \\ \mathbf{u}^i \geq 0 \end{array}\right\} \; i = 1, \ldots, m.
\end{aligned}
$$

- Some objective terms and constraints drop out of the formulation

Deriving The Hypothesis Update

- First sub-problem: update hypothesis by fixing the advice vectors

$$
\begin{aligned}
\mathbf{w}^{t+1} \;=\; \min_{\mathbf{w},\xi,\eta^i} \quad & \frac{1}{2}\|\mathbf{w}-\mathbf{w}^t\|_2^2 + \frac{\lambda}{2}\xi^2 + \frac{\mu}{2}\sum_{i=1}^{m}\|\eta^i\|_2^2 \\
\text{subject to} \quad & y_t\mathbf{w}'\mathbf{x}^t - 1 + \xi \geq 0, & (\alpha) \\
& D_i'\mathbf{u}^{i,t} + z_i\mathbf{w} + \eta^i = 0,\; i = 1,\ldots,m. & (\beta^i)
\end{aligned}
$$

Deriving The Hypothesis Update

- First sub-problem: update hypothesis by fixing the advice vectors

$$\begin{aligned} \mathbf{w}^{t+1} &= \min_{\mathbf{w},\xi,\eta^i} \quad \frac{1}{2}\|\mathbf{w}-\mathbf{w}^t\|_2^2 + \frac{\lambda}{2}\xi^2 + \frac{\mu}{2}\sum_{i=1}^{m}\|\eta^i\|_2^2 \\ &\text{subject to} \quad y_t\mathbf{w}'\mathbf{x}^t - 1 + \xi \geq 0, && (\alpha) \\ &\qquad\qquad D_i'\mathbf{u}^{i,t} + z_i\mathbf{w} + \eta^i = 0,\ i = 1,\ldots,m. && (\beta^i) \end{aligned}$$

fixed, ***advice-estimate of the hypothesis according to i-th advice set*** ; denote as $\mathbf{r}^{i,t}$

Advice-Estimate Of Current Hypothesis

- First sub-problem: update hypothesis by fixing the advice vectors

$$\mathbf{w}^{t+1} = \min_{\mathbf{w},\xi,\eta^i} \quad \frac{1}{2}\|\mathbf{w}-\mathbf{w}^t\|_2^2 + \frac{\lambda}{2}\xi^2 + \frac{\mu}{2}\sum_{i=1}^{m}\|\eta^i\|_2^2$$

$$\text{subject to} \quad y_t\mathbf{w}'\mathbf{x}^t - 1 + \xi \geq 0, \qquad (\alpha)$$

$$D_i'\mathbf{u}^{i,t} + z_i\mathbf{w} + \eta^i = 0, \; i = 1, \ldots, m. \qquad (\beta^i)$$

fixed, ***advice-estimate of the hypothesis according to i-th advice set*** ; denote as $\mathbf{r}^{i,t}$

average ***advice-estimates over all m advice vectors*** and denote as

$$\mathbf{r}^t = \frac{1}{m}\sum_{i=1}^{m}\mathbf{r}^{i,t}$$

The Hypothesis Update

For $\lambda,\ \mu > 0$, and advice-estimate $\mathbf{r}^t$, the hypothesis update is

$$\mathbf{w}^{t+1} \; = \; \nu\left(\mathbf{w}^t + \alpha_t y_t \mathbf{x}^t\right) + (1-\nu)\,\mathbf{r}^t,$$

$$\alpha_t = \left(\frac{1}{\lambda} + \nu\|\mathbf{x}^t\|^2\right)^{-1} \cdot \max\left(\, 1 - \nu\, y_t {\mathbf{w}^t}' \mathbf{x}^t - (1-\nu)\, y_t {\mathbf{r}^t}' \mathbf{x}^t \;,\; 0\right).$$

The Hypothesis Update

For λ, $\mu > 0$, and advice-estimate $\mathbf{r}^t$, the hypothesis update is

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$$\alpha_t = \left(\frac{1}{\lambda} + \nu\|\mathbf{x}^t\|^2\right)^{-1} \cdot \max\left(1 - \nu\, y_t \mathbf{w}^{t\prime} \mathbf{x}^t - (1-\nu)\, y_t \mathbf{r}^{t\prime} \mathbf{x}^t\ ,\ 0\right).$$

Update is ***convex combination*** of the standard ***passive-aggressive update*** and the ***average advice-estimate***

Parameter of convex combinations is $\nu = \dfrac{1}{1+m\mu}$

The Hypothesis Update

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Update is ***convex combination*** of the standard ***passive-aggressive update*** and the ***average advice-estimate***

Update weight depends on ***hinge loss*** computed with respect to a ***composite weight vector*** that is a ***convex combination*** of the ***current hypothesis*** and the ***average advice-estimate***

Parameter of convex combinations is $\nu = \dfrac{1}{1+m\mu}$

Deriving The Advice Updates

- Second sub-problem: update advice vectors by fixing the hypothesis $\mathbf{w} = \mathbf{w}^{t+1}$

$$
\begin{aligned}
\min_{\xi, \mathbf{u}^i, \eta^i, \zeta_i \geq 0, \mathbf{w}} \quad & \frac{1}{2}\|\mathbf{w} - \mathbf{w}^t\|^2 + \frac{1}{2}\sum_{i=1}^{m} \|\mathbf{u}^i - \mathbf{u}^{i,t}\|^2 + \frac{\lambda}{2}\xi^2 + \frac{\mu}{2}\sum_{i=1}^{m}\left(\|\eta^i\|^2 + \zeta_i^2\right), \\
\text{subject to} \quad & y_t \mathbf{w}'\mathbf{x}^t - 1 + \xi \geq 0, \\
& \left.\begin{aligned}
D_i'\mathbf{u}^i + z_i \mathbf{w} + \eta^i &= 0 \\
-\mathbf{d}^{i\prime}\mathbf{u}^i - 1 + \zeta_i &\geq 0 \\
\mathbf{u}^i &\geq 0
\end{aligned}\right\} \; i = 1, \ldots, m.
\end{aligned}
$$

- Some constraints and objective terms drop out of the formulation

Deriving The Advice Updates

- Second sub-problem: update advice vectors by fixing the hypothesis

$$
\begin{array}{rl}
\displaystyle\min_{\mathbf{u}^i,\eta^i,\zeta_i\geq 0,} & \displaystyle\frac{1}{2}\sum_{i=1}^{m}\|\mathbf{u}^i-\mathbf{u}^{i,t}\|^2+\frac{\mu}{2}\sum_{i=1}^{m}\left(\|\eta^i\|^2+\zeta_i^2\right), \\
\text{subject to} & \left.\begin{aligned} D_i'\mathbf{u}^i+z_i\mathbf{w}^{t+1}+\eta^i&=0\\ -\mathbf{d}^{i'}\mathbf{u}^i-1+\zeta_i&\geq 0\\ \mathbf{u}^i&\geq 0\end{aligned}\right\}\ i=1,\ldots,m.
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Deriving The Advice Updates

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\end{array}
$$

split into *m* sub-problems

Deriving The i-th Advice Updates

- *m* sub-problems: update the *i*-th advice vector by fixing the hypothesis

$$
\begin{aligned}
\mathbf{u}^{i,t+1} = \min_{\mathbf{u}^i,\eta,\zeta} \quad & \frac{1}{2}\|\mathbf{u}^i - \mathbf{u}^{i,t}\|^2 + \frac{\mu}{2}\left(\|\eta^i\|_2^2 + \zeta_i^2\right) \\
\text{subject to} \quad & D_i'\mathbf{u}^i + z_i\mathbf{w}^{t+1} + \eta^i = 0, && (\beta^i) \\
& -\mathbf{d}^{i\prime}\mathbf{u}^i - 1 + \zeta_i \geq 0, && (\gamma_i) \\
& \mathbf{u}^i \geq 0. && (\tau^i)
\end{aligned}
$$

Deriving The i-th Advice Updates

- *m* sub-problems: update the *i*-th advice vector by fixing the hypothesis

$$
\begin{aligned}
\mathbf{u}^{i,t+1} = \min_{\mathbf{u}^i,\eta,\zeta} \quad & \frac{1}{2}\|\mathbf{u}^i - \mathbf{u}^{i,t}\|^2 + \frac{\mu}{2}\left(\|\eta^i\|_2^2 + \zeta_i^2\right) \\
\text{subject to} \quad & D_i'\mathbf{u}^i + z_i\mathbf{w}^{t+1} + \eta^i = 0, && (\beta^i) \\
& -\mathbf{d}^{i\prime}\mathbf{u}^i - 1 + \zeta_i \geq 0, && (\gamma_i) \\
& \mathbf{u}^i \geq 0. && (\tau^i)
\end{aligned}
$$

cone constraints still complicating

cannot derive closed form solution

- ***Use projected-gradient approach***
  - ***drop constraints*** to compute intermediate closed-form update
  - ***project*** intermediate update back on to cone constraints

The *m* Advice Updates

For $\mu > 0$, and current hypothesis $\mathbf{w}^{t+1}$, for each advice set, $i = 1, \ldots, m$, the update rule is given by

$$\mathbf{u}^{i,t+1} \;=\; \max\left(\mathbf{u}^{i,t} + D_i\beta^i - \mathbf{d}^i\,\gamma_i,\; 0\right),$$

$$\begin{bmatrix} \beta^i \\ \gamma_i \end{bmatrix} = \begin{bmatrix} -(D_i'D_i + \frac{1}{\mu}I_n) & D_i'\mathbf{d}^i \\ \mathbf{d}^{i\prime}D_i & -(\mathbf{d}^{i\prime}\mathbf{d}^i + \frac{1}{\mu}) \end{bmatrix}^{-1} \begin{bmatrix} D_i'\mathbf{u}^{i,t} + z_i\mathbf{w}^{t+1} \\ -\mathbf{d}^{i\prime}\mathbf{u}^{i,t} - 1 \end{bmatrix}$$

The *m* Advice Updates

For $\mu > 0$, and current hypothesis $\mathbf{w}^{t+1}$, for each advice set, $i = 1, \ldots, m$, the update rule is given by

$$\mathbf{u}^{i,t+1} = \max\left(\mathbf{u}^{i,t} + D_i\beta^i - \mathbf{d}^i\,\gamma_i,\ 0\right),$$

projection

$$\begin{bmatrix} \beta^i \\ \gamma_i \end{bmatrix} = \begin{bmatrix} -(D_i'D_i + \frac{1}{\mu}I_n) & D_i'\mathbf{d}^i \\ \mathbf{d}^{i\prime}D_i & -(\mathbf{d}^{i\prime}\mathbf{d}^i + \frac{1}{\mu}) \end{bmatrix}^{-1} \begin{bmatrix} D_i'\mathbf{u}^{i,t} + z_i\mathbf{w}^{t+1} \\ -\mathbf{d}^{i\prime}\mathbf{u}^{i,t} - 1 \end{bmatrix}$$

- ***hypothesis-estimate of the advice;*** denote $\mathbf{s}^i = \beta^i/\gamma_i$
- The update is the ***error*** or the amount of ***violation of the constraint*** $D_i\mathbf{x} \leq \mathbf{d}^i$ by an ideal data point, $\mathbf{s}^i$

each advice update ***depends on the newly updated hypothesis***

The Adviceptron

input: data $(\mathbf{x}^t, y_t)_{t=1}^T$, advice sets $(D_i, \mathbf{d}^i, z_i)_{i=1}^m$, parameters λ, $\mu > 0$

initialize: $\mathbf{u}^{i,1} = 0$, $\mathbf{w}^1 = 0$

let: $\nu = 1/(1 + m\mu)$

for $(\mathbf{x}^t,\, y_t)$ **do**

predict label $\hat{y}_t = \mathrm{sign}(\mathbf{w}^{t\prime}\mathbf{x}^t)$

receive correct label y_t

suffer loss

$$\ell_t = \max\left(1 - \nu y_t \mathbf{w}^{t\prime}\mathbf{x}^t - (1-\nu) y_t \mathbf{r}^{t\prime}\mathbf{x}^t,\ 0\right)$$

update hypothesis using $\mathbf{u}^{i,t}$

$$\alpha = \ell_t / (\frac{1}{\lambda} + \nu \|\mathbf{x}^t\|^2), \quad \mathbf{w}^{t+1} = \nu\,(\,\mathbf{w}^t + \alpha\, y_t \mathbf{x}^t\,) + (1-\nu)\,\mathbf{r}^t$$

update advice variables using $\mathbf{w}^{t+1}$

$$(\beta^i,\, \gamma_i) = H_i^{-1}\mathbf{g}^i, \quad \mathbf{u}^{i,t+1} = \left(\mathbf{u}^{i,t} + D_i\beta^i - \mathbf{d}^i\gamma_i\right)_+$$

end for

Outline

- Knowledge-Based Support Vector Machines
- The Adviceptron: Online KBSVMs
- ***A Real-World Task: Diabetes Diagnosis***
- A Real-World Task: Tuberculosis Isolate Classification
- Conclusions

Diagnosing Diabetes

- Standard data set from UCI repository (768 x 8)
  - all patients at least 21 years old of Pima Indian heritage
  - features include body mass index, blood glucose level
- Expert advice for diagnosing diabetes from NIH website on risks for Type-2 diabetes
  - a person who is obese (characterized by BMI > 30) and has a high blood glucose level (> 126) is at a strong risk for diabetes

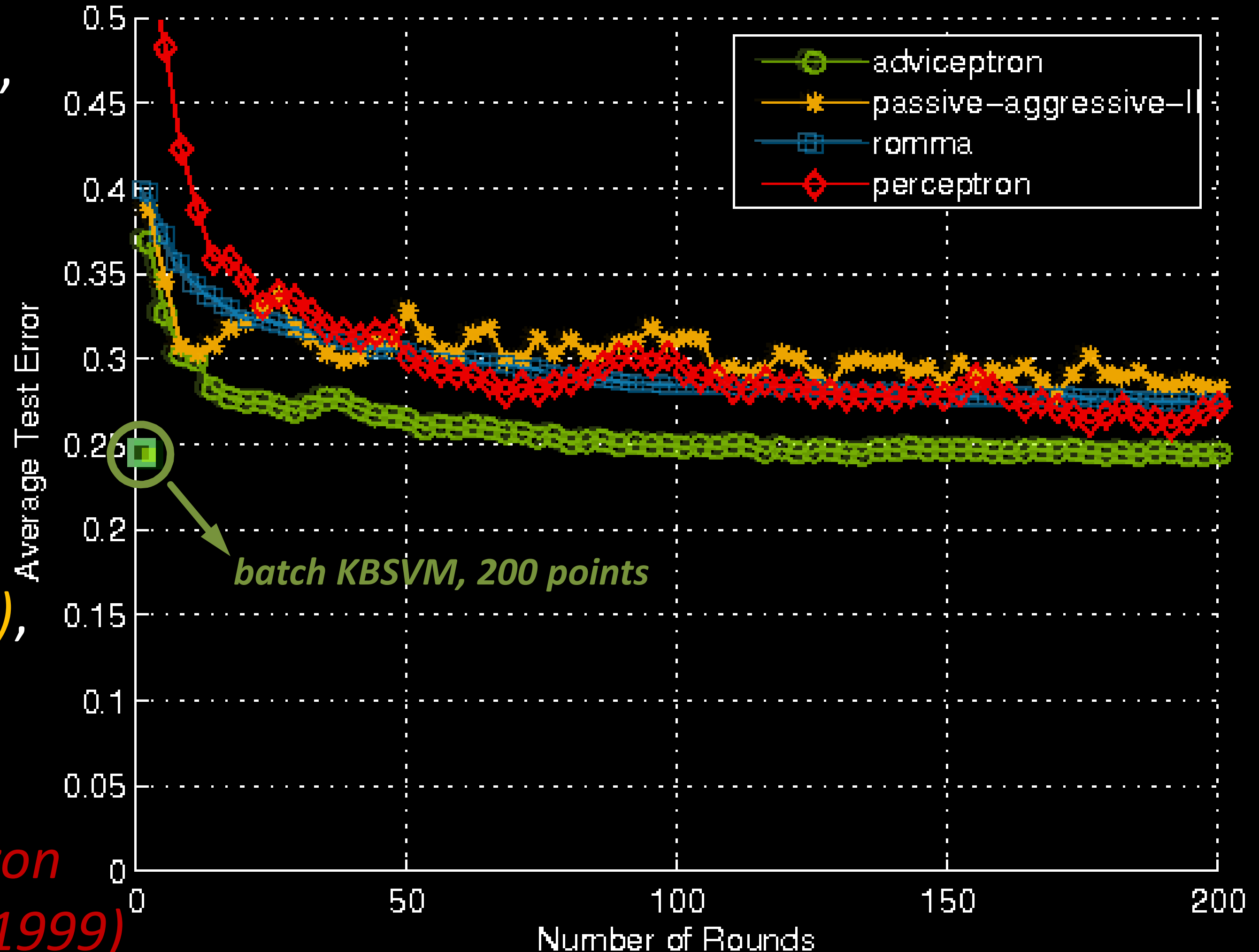
$$(\mathtt{BMI} \geq 30) \wedge (\mathtt{bloodglucose} \geq 126) \Rightarrow \mathtt{diabetes}$$

  - a person who is at normal weight (BMI < 25) and has low blood glucose level (< 100) is at a low risk for diabetes

$$(\mathtt{BMI} \leq 25) \wedge (\mathtt{bloodglucose} \leq 100) \Rightarrow \neg\mathtt{diabetes}$$

Diagnosing Diabetes: Results

- 200 examples for training, remaining for testing
- Results averaged over 20 randomized iterations
- Compared to advice-free online algorithms:
  - *Passive-aggressive (Crammer et al, 2006)*,
  - *ROMMA (Li & Long, 2002)*,
  - *Max margin-perceptron (Freund & Schapire, 1999)*



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Tuberculosis Isolate Classification

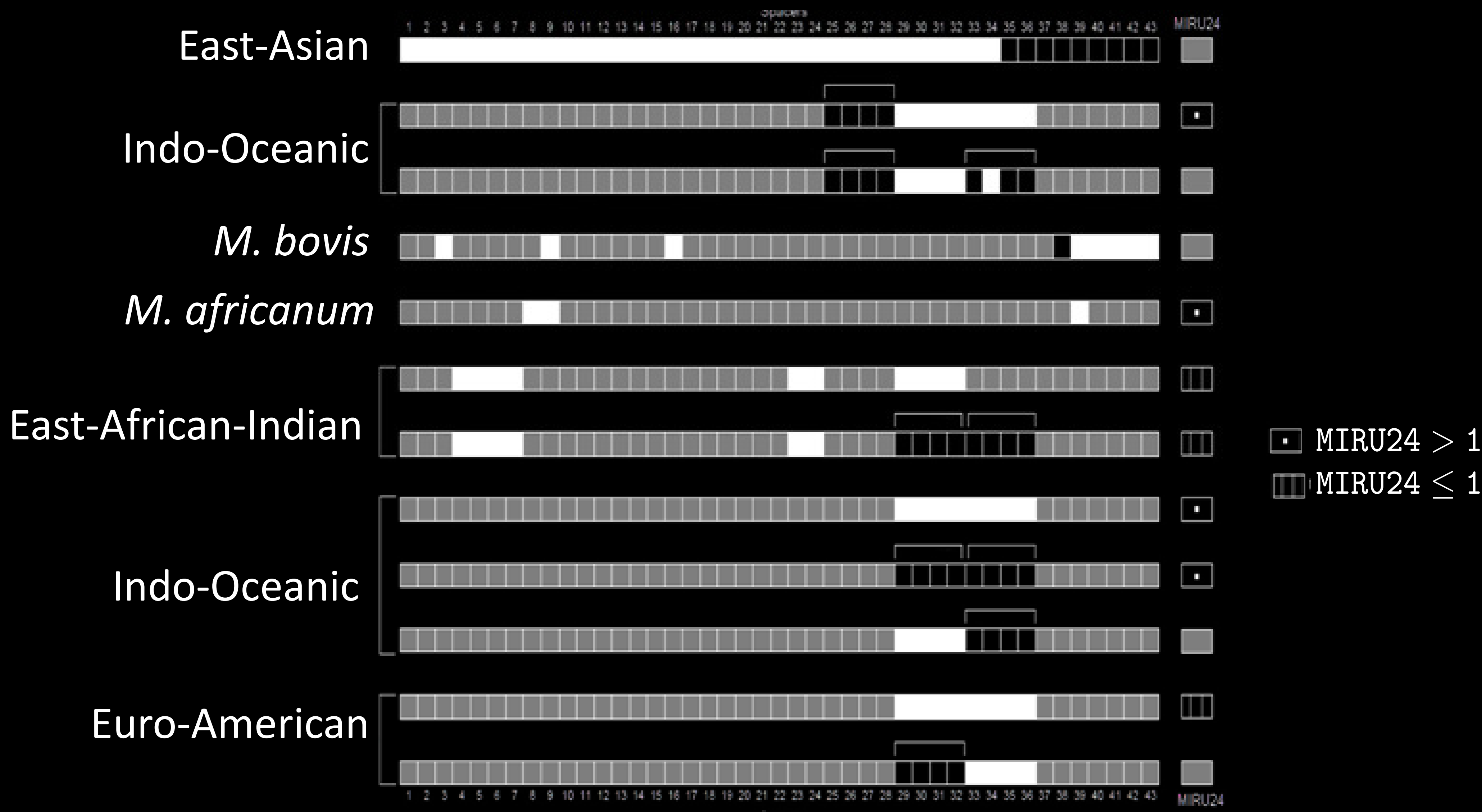
- Task is to classify strains of *Mycobacterium tuberculosis* complex (MTBC) into major genetic lineages based on DNA fingerprints
- MTBC is the causative agent for TB
  - leading cause of disease and morbidity
  - strains vary in infectivity, transmission, virulence, immunogenicity, host associations depending on genetic lineage
- Lineage classification is crucial for surveillance, tracking and control of TB world-wide

Tuberculosis Isolate Classification

- Two types of DNA fingerprints for all culture-positive TB strains collected in the US by the CDC (44 data features)
- Six (classes) major lineages of TB for classification
  - ***ancestral:*** *M. bovis, M. africanum,* Indo-Oceanic
  - ***modern:*** Euro-American, East-Asian, East-African-Indian
- Problem formulated as six 1-vs-many classification tasks

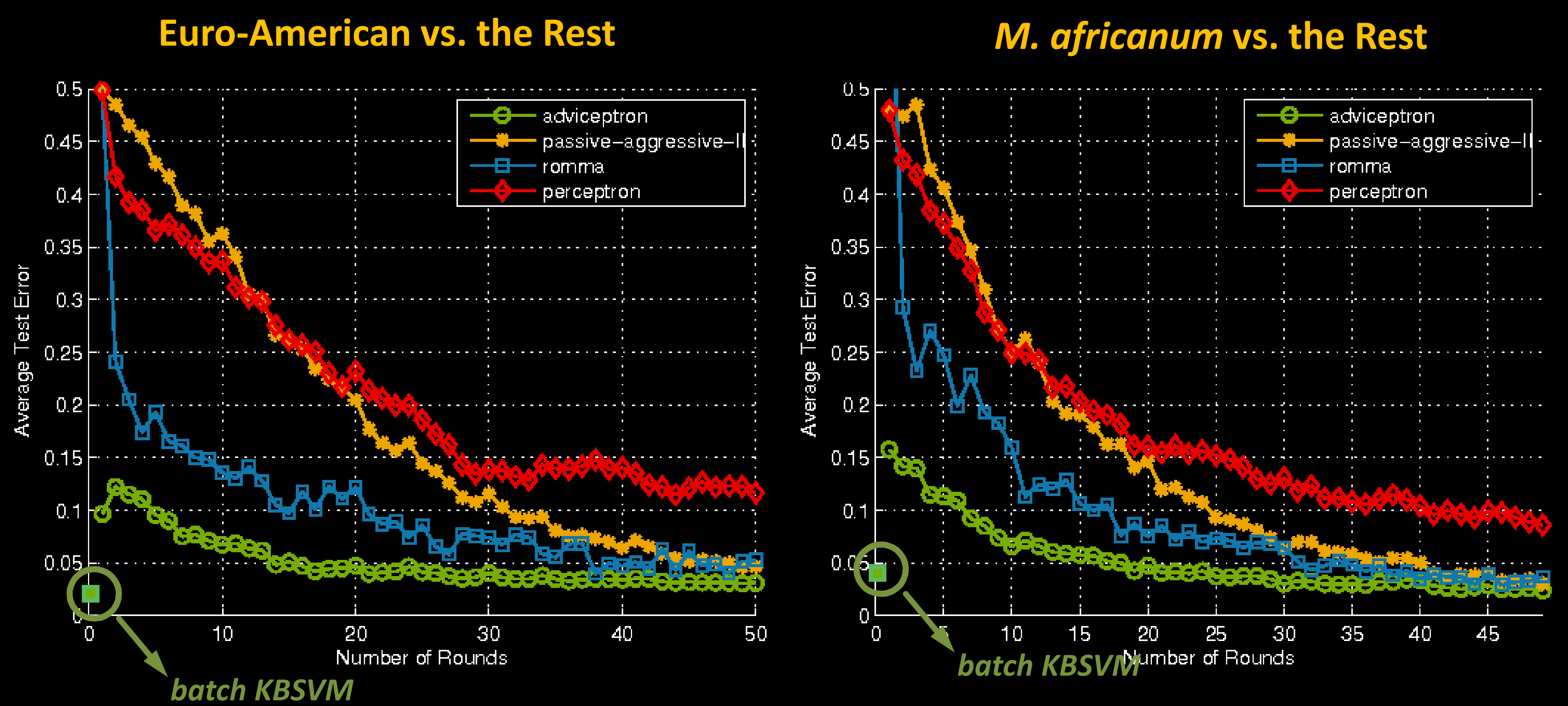
Class	#isolates	#pieces of Positive Advice	#pieces of Negative Advice
East-Asian	4924	1	1
East-African-Indian	1469	2	4
Euro-American	25161	1	2
Indo-Oceanic	5309	5	5
M. africanum	154	1	3
M. bovis	693	1	3

Expert Rules for TB Lineage Classification

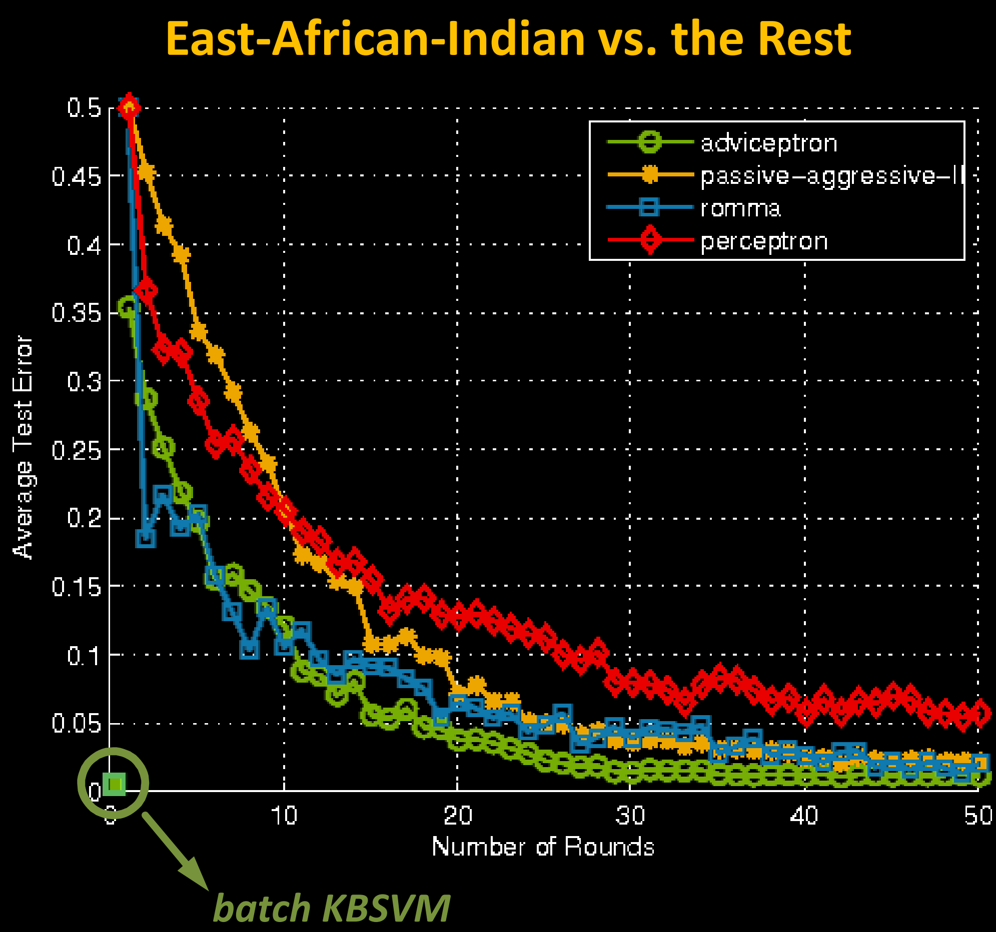


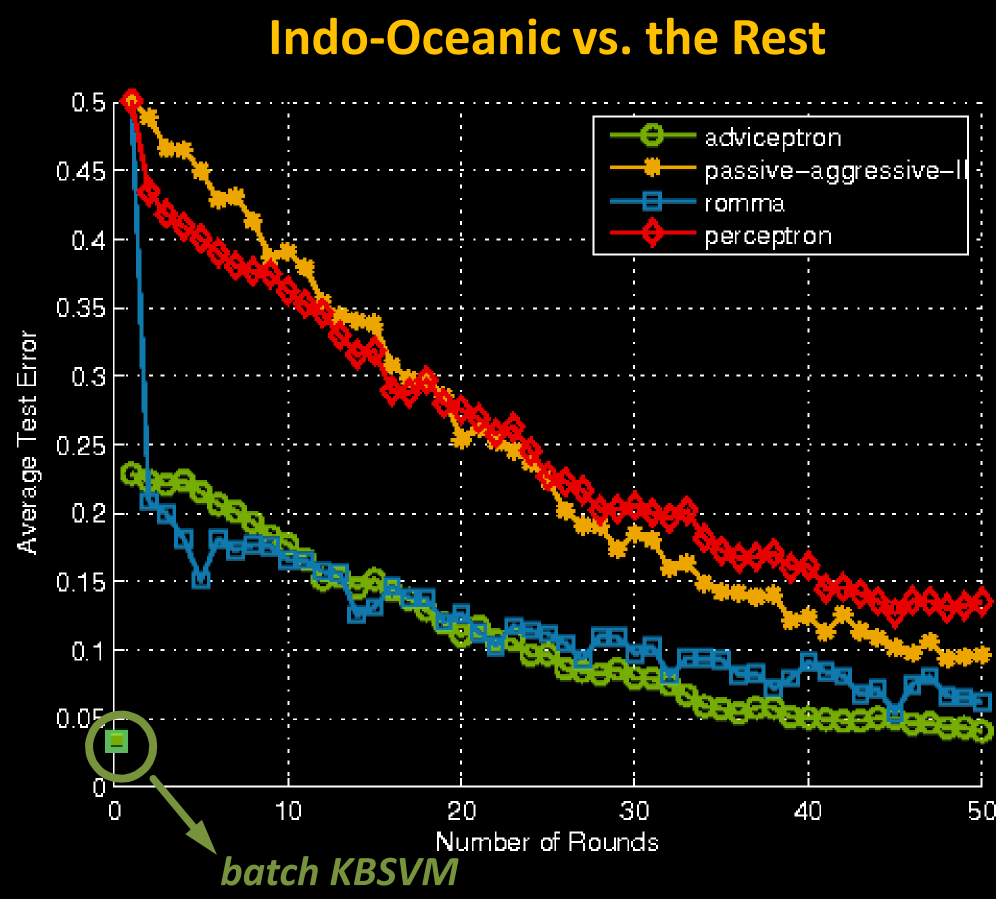
Rules provided by Dr. Lauren Cowan at the Center for Disease Control, documented in Shabbeer et al, (2010)

TB Results: Might Need Fewer Examples To Converge With Advice



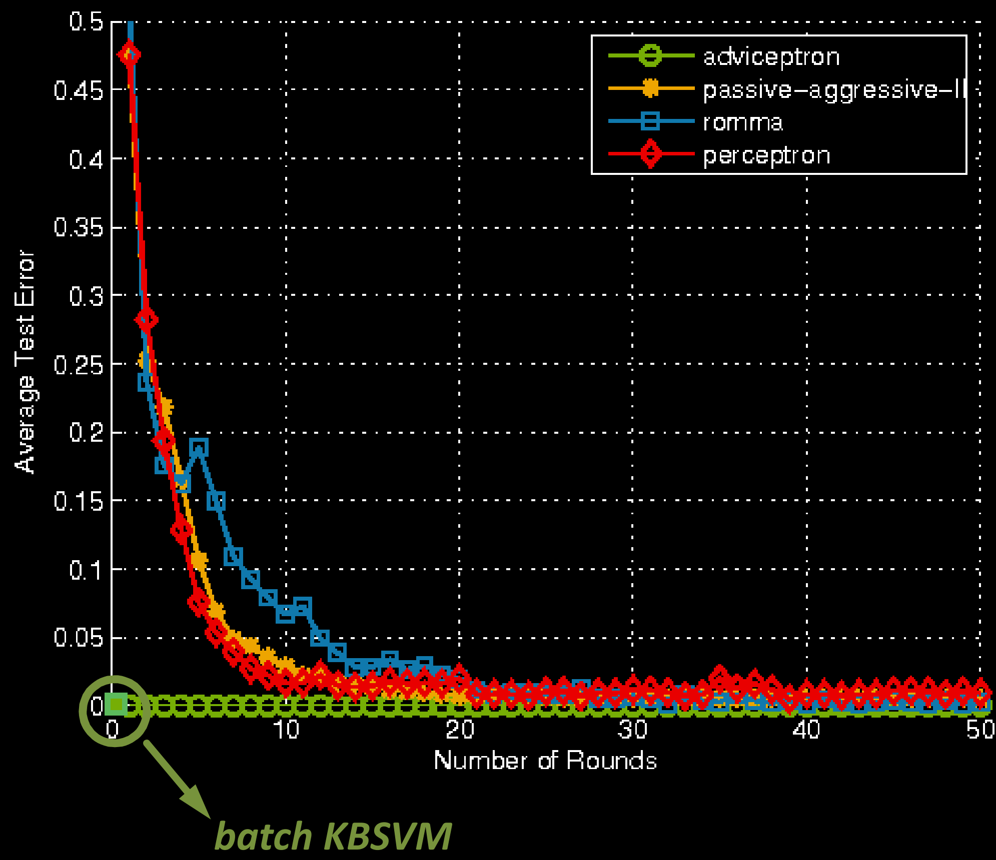
TB Results: Can Converge To A Better Solution With Advice



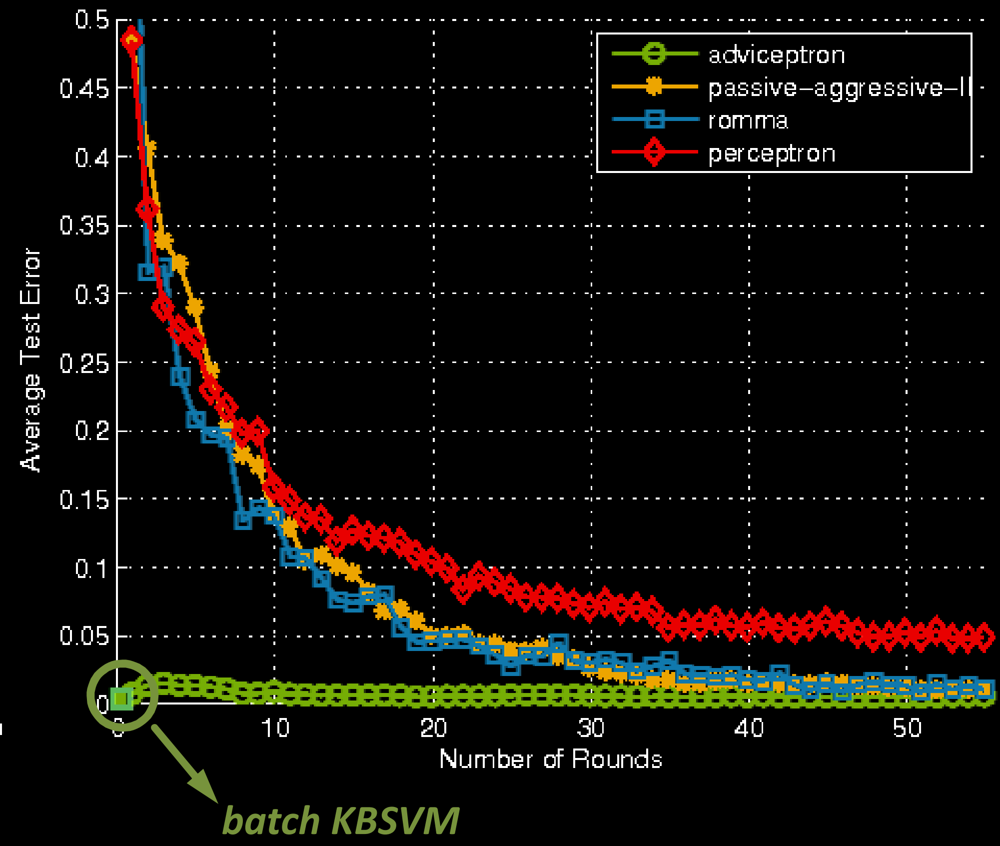


TB Results: Possible To Still Learn Well With ***Only*** Advice

East-Asian vs. the Rest



M. bovis **vs. the Rest**



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- ***Conclusions And Questions***

Conclusions

- New online learning algorithm: ***the adviceptron***
- Makes use of prior knowledge in the form of (possibly imperfect) ***polyhedral advice***
- Performs ***simple, closed-form updates*** via passive-aggressive framework; scalable
- Good advice can help converge to a ***better solution with fewer examples***
- ***Encouraging empirical results*** on two important real-world tasks

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KBSVMs: Deriving The Advice Constraints

We assume an expert provides polyhedral advice of the form

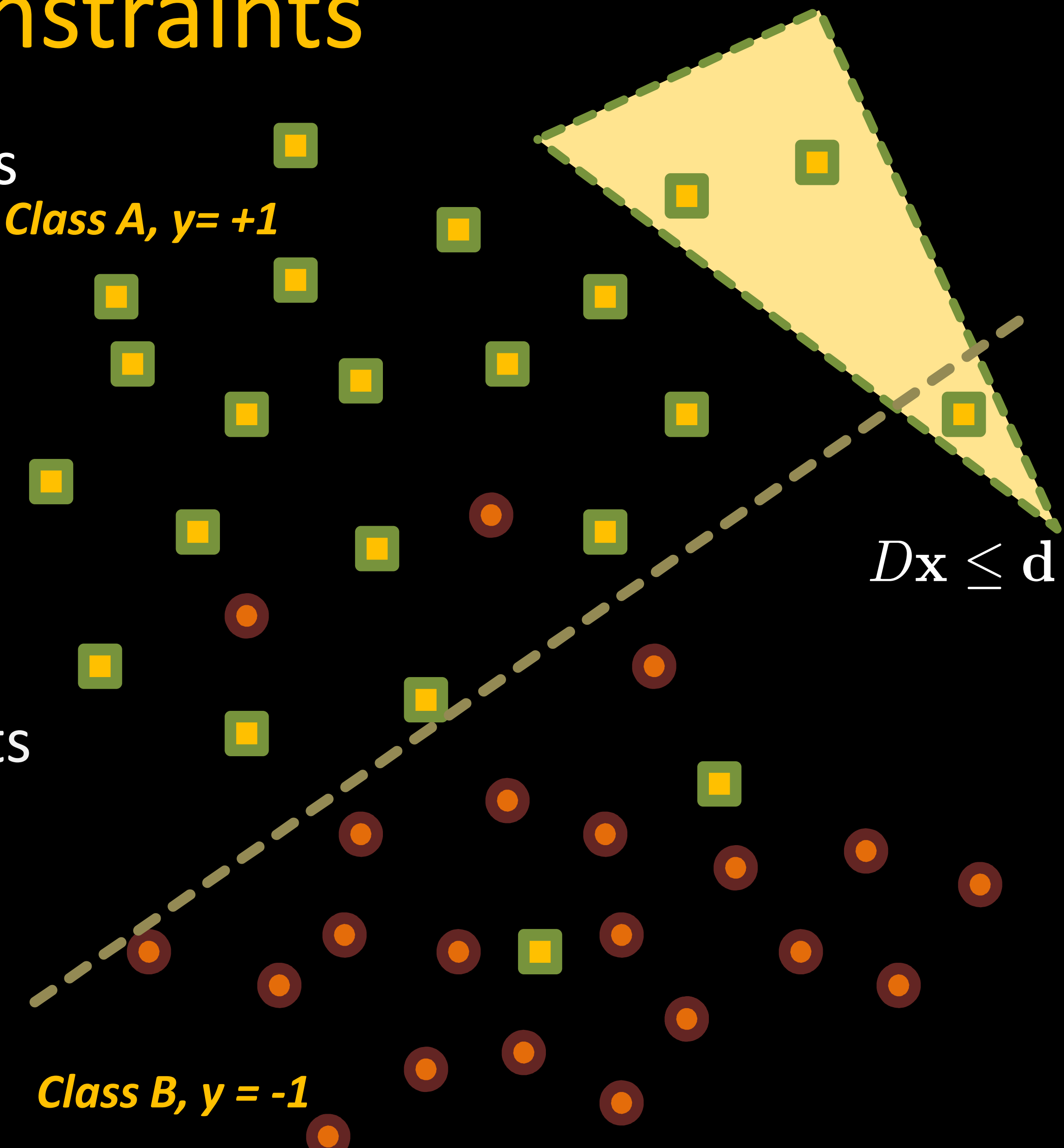
$$D\mathbf{x} \leq \mathbf{d} \Rightarrow \mathbf{w}'\mathbf{x} \geq b$$

We know $p \Rightarrow q$ is equivalent to $\neg p \vee q$

If $\neg p \vee q$ has a solution then its **negation has no solution** or,

$$\begin{aligned} D\mathbf{x} - \mathbf{d}\,\tau &\leq 0, \\ \mathbf{w}'\mathbf{x} - b\,\tau &< 0, \\ -\tau &< 0 \end{aligned}$$

has no solution $(\mathbf{x}, \tau)$.



KBSVMs: Deriving The Advice Constraints

If the following system

$$\begin{aligned} D\mathbf{x} - \mathbf{d}\,\tau &\leq 0, \\ \mathbf{w}'\mathbf{x} - b\,\tau &< 0, \\ -\tau &< 0 \end{aligned}$$

has no solution $(\mathbf{x}, \tau)$, then by ***Motzkin's Theorem of the Alternative***, the following system

$$\begin{aligned} D'\mathbf{u} + \mathbf{w} &= 0, \\ -\mathbf{d}'\mathbf{u} - b &\geq 0, \\ \mathbf{u} &\geq 0 \end{aligned}$$

has a solution u.