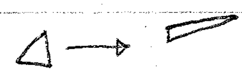
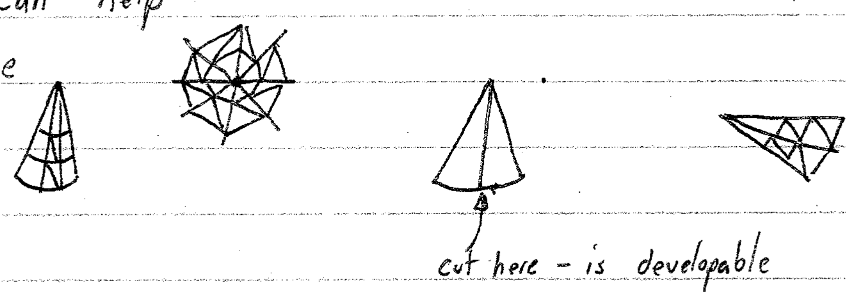
Parameterizations

What is good ?

- Avoid Local Distortions 
  length, angles, areas
- 1 - to - 1
- avoid cuts and discontinuities
- easy / efficient
- good paching (use all of texture space)
- planar models preserved - Linear Reproduction

Cuts can help -

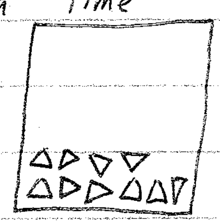
Cone



NAIVE Algorithm

Triangles 1 at a time

Paching problem



No distortion!

Lots of cuts

Why is this bad ?

Can easily come up with other variants

Greedily place triangles

Chameleon painting system (2000)

When painted, screen coords = tex coords

assign lazily

When rotate - move strokes to texture map - paching

(what if paint same triangle twice ?)

What's good - easy to implement

works well w/ screen space drawing tools

puts seams in places they don't matter

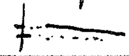
only parameterizes stuff you paint

What's bad (why is exp map better)

ignores invisible stuff (foldovers leave holes)

screen space may not be good parameterization

oblique = few pixels big triangle (who cares - little paint)

 ← can't put lots of detail anyway

distortions mean textures hard to edit (in texture space)

sampling issues

Parameterizing topological discs

Set of triangles with an edge

Two strategies
- fixed resulting edge shape
- free edge shape (figures out the best edge shape)

Fixed Edge

Clearly can distort if edge is bad

$\bigcirc \to \square$

Has easier methods

Sometimes you want to fill domain (every point on texture goes somewhere)

Points assigned positions on edge

Mass spring system

vertices are points.

connect edges w/ springs - choose spring constant / rest length somehow (K)

find minimum energy configuration

energy is quadratic (if zero rest length)

find minimum by solving a linear system

Cool theory - planar graph embedding

for just about any reasonable choice of K_s, graph will be planar

What choice of K?

$K = 1$ for all springs (Tutte embedding for planar graphs)

- doesn't preserve planar things

Spring: $E = \frac{1}{2} k \|a - b\|^2$

D is matrix of k's

$$E = \sum \frac{1}{2} D_{ij} \|v_i - v_j\|^2$$

$$\frac{\partial E}{\partial v_i} = \sum D_{ij} (v_i - v_j)$$

$$v_i = \sum_{j \in N_i} \lambda_{ij} v_j$$

λ_{ij}: normalized row row D made to sum to 1

Big linear system $Av = 0$ (find minimum)

put known points on RHS (constrained optimum)

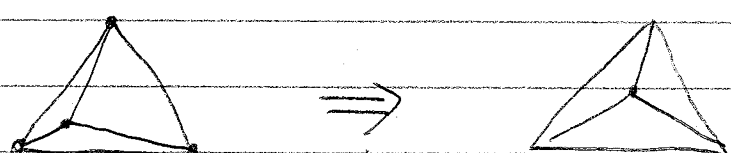
$$A v_x = \bar{v}_x \quad \text{(can do x and y seperately)}$$

$$a_{ij} = \begin{cases} 1 & i = j \\ -\lambda & \text{if } i \neq j,\ j \in N_i \\ 0 & \text{otherwise} \end{cases} \qquad \bar{v}_i = \sum_{j \in N_i,\ j \text{ on edge}} \lambda_{ij} v_j$$

v_j: known fixed position

See SIGCOURSE 08 p 7-9 sec 2.2 - very nicely explained!

What about Linear Reproduction?



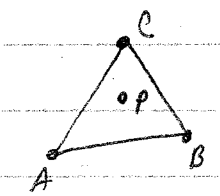
constant weights

Idea :

Coordinates of point in its neighborhood polygon

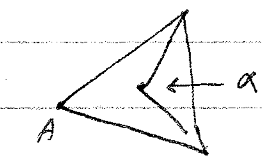
Barycentric Coordinates

Each point is a linear combination of the "basis" points



$p = \alpha A + \beta B + \gamma C$

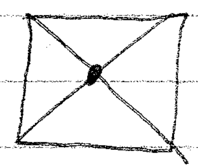
related to area



What if polygon is > than 3 sides?

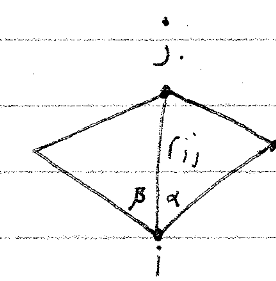
NOT UNIQUE

Generalized Barycentric coords



Uses : interpolation / warping

smoother than triangulation



Variants of Genealized BC

- Watchpress Coords
- Discrete Harmonic Coords
- Mean-Value (not discovered until 2003)

$$W_{ij} = \frac{\tan\frac{\alpha_{ij}}{2} + \tan\frac{\beta_{ij}}{2}}{r_{ij}}$$

MVC - always positive (no foldovers for parameterization)

still pretty simple

Beyond fixed boundary
Aside - WHY DID WE LIKE THESE METHODS
"GLOBAL" ← spread problems around uniformly
considered all triangles at once

Idea - optimization problem
minimize distortion summed over all triangles
pick a distortion metric that is easy to minimize
quadratic function (so solve a linear system)

Least - Squares Conformal Maps
requires at least 2 points to be specified
(otherwise, all points @ origin = no distortion)

interested in the singular values of local map
(stretches in principle directions)
expressions give quadratic forms
- solve a linear system

Angle-Based Flattening
work in angle space only
- sufficient up to similarity
Constain triangles to be triangles $\Sigma \alpha^T = \pi$
to be planar (around point = 2π) $\Sigma \alpha_i = 2\pi$
then try to match original model's ∡ as close as possible
Needs a few improvements (ABF++)