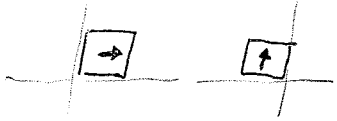
ROTATIONS

TRIVIAL Example 

$$R_{90} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Half-Way

$\frac{1}{2} R_{90}$ $\qquad \frac{1}{2}(R_0 + R_{90})$

↑ is identity, not zero!

$R_{45} =$

ROTATION vs. Orientation
- halfway doesn't make sense
- multiple spins ↑ halfway between equivalences 2 things
- rotations between 2 orientations

Transformation Composition

$M_{AB} = M_A M_B$

$R_{90} = R_{45} R_{45}$ $\qquad R_{45} = \sqrt{R_{90}}$

Rotation vs Motion? Just end result?

Log Space?

$AB = e^{\ln(AB)}$

$\ln e^{AB} = e^A e^B = e^{(A+B)}$?

not exactly true for matrices
+ commutes
* does not

Interpolation $A \to B$ $\qquad A(A^{-1}B)^t \qquad t \in 0,1$

$\Rightarrow e^{(1-t)\log A + t \log B}$

What is a rotation anyway?
Rigid Transform (distance preserving)
With a center, preserves handedness

$f(0) = 0$
$\|A - B\| = \|f(A) - f(B)\|$

must be an ortho-normal matrix

Set of rotations = Set of Funktions = Set of Matrices
↑ all nxn matrices
$R^T R = I$, $\det(R) = 1$

GROUP (over operator $\circ$)
1. closure $A \circ B \in G$
2. identity $I \circ A = A$
3. inverse $A \circ A^{-1} = I$ all $A \in g$ have A^{-1}
4. associative

NOT COMMUTATIVE

WAIT - DOES A MATRIX ENCODE ALL ROTATIONS?
sortof → many → one mapping
rot matrix is a parameterization

Parameterizations

The "things" (rotations)

vs.

Their name / representation

> if we're going to compute with them, we need something (true in general)

What do you want to do with a rotation - different parameterizations have different pros/cons

2D - ways to describe the set

① Circle (where 1,0 goes) ← 1 point in 2D

⇒ complex numbers

NOTE THE "SHAPE" OF The Space circle is not a line

② Distance around the circle

③ Velocity of a point (unit time interval, constant velocity magnitude)

axis of rotation × position = direction

doesn't need to be origin

④ Matrix (subset)

downsides: redundant hard to operate on (interp, ...)

drift