

# SUBDIVISION SURFACES.

11/27/01

## Announcements :

Project 2 Demo Signups

Quiz Grading

Project 3

Readings

Texture Lecture - read Shirley and Haines; Moller

## Today -

### Subdivision Surfaces

- not a new idea

Catmull - Clark : Boo-Babin late 70s

Mathematics long before that

- came back to graphics  $\approx$  1998

A few breakthroughs

A few high profile successes

Pent-Up frustrations with Alternatives

## What happened in 1998?

Ger's game - PIXAR

paper by DeRose, Huss, and Truong  
shows you can do everything with  
subdivision

Joe Stam - Exact Evaluation methods  
shows that can be efficient for just  
about anything

Why Subdivision

- Alternative: Tensor Product Surfaces  
Polygons

Why not NURBS? (remember 2 lectures ago?)

- fixed Topology
- can't stitch together
- can't trim
- can't get sharp creases
- can't get more detail locally
- how to get nice parameterizations?

That's a high price to pay! what do we get?

- smoothness
- parametric - have coordinates (for texturing)  
even if "weird"
- compute exactly - know tangents and positions  
for all points

Why not just use polygons?

- not smooth

Use more polygons to get smoothness?

as you add more and more polygons,  
gets closer and closer to being smooth

(3)

How to get more polygons?

make a rule about how to compute given  
current set

Subdivision:

initial shape

rule to divide each element into smaller pieces

apply rule over and over until done

Example: (2d points, curves)

given points  $i, i+1$ , insert new point  $\frac{P_i + P_{i+1}}{2}$

• •  $\Rightarrow$  ...  $\Rightarrow$  ....  $\Rightarrow$

in the limit this shape is continuous

Example:

Quadratic Bezier Curves (from Study p. 208)  
(remember, Bezier's interpolate end points)

Given:  $P_0, P_1, P_2$

move  $P_1$ , insert  $P_{.5}, P_{1.5}$

$$P_{.5} = \frac{1}{2}(P_0 + P_1)$$

$$P_{1.5} = \frac{1}{2}(P_1 + P_2)$$

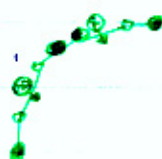
$$P_1' = \frac{1}{2}(P_{.5} + P_{1.5})$$



now subdivide

$P_0, P_{.5}, P_1$

$P_1, P_{1.5}, P_2$



④

IN THE LIMIT WE GET A SMOOTH CURVE!

The problems:

① What should the rules be  
(the "subdivision scheme")

How can we be sure we get something that converges?

② How much do we need to subdivide?

③ What if we don't want to do  $\infty$  subdivision steps?

④ How to deal with different levels of detail?  
(some close together points, some far away points)

## SUBDIVISION SCHEMES

Many Different ones - find one that fits your needs  
interpolating vs. approximating  
triangles vs. quads  
how are "extra-ordinary" points handled?

Support: the set of points that influence the  
new point position computation

# EXAMPLE 1 : BUTTERFLY SCHEME

## INTERPOLATING

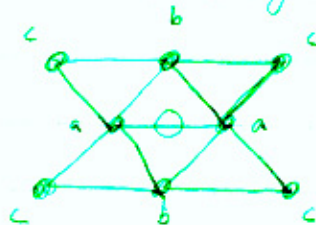
C1 "almost everywhere"

Triangles: Split Edges



1 Triangle  $\Rightarrow$  4

new position = weighted average of 8 neighbors



$$a = \frac{1}{2} \quad b = \frac{1}{8} \quad c = -\frac{1}{16}$$

What about points that have more or less than 8 neighbors?

Need special rules! (see papers)

Where are points on the limit surface?

Easy! - since the surface interpolates, positions of the controls or current mesh = position of point on limit surface

What are the normals?

can't subdivide for ever!

no problem - have a formula (a different "mesh")

6

Butterfly details in Gamasutra article

Why not Butterfly?

C1  $\leftarrow$  and not at extraordinary points  
modified rules do better

"ugly"

interpolating curves / surfaces never look as good

Loop Scheme —

Popular

Approximating

Triangles

(each edge gets split)

⑦

# Catmull - Clark surfaces

## Approximating Scheme

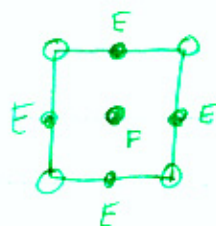
Generally on Quads - breaks <sup>non-</sup>quads into ~~non~~ quads

generalizes cubic B-splines -

gives same shapes!

can convert B-splines  $\Rightarrow$  CAT-Clark

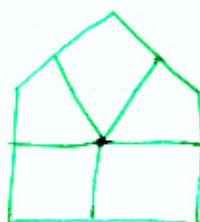
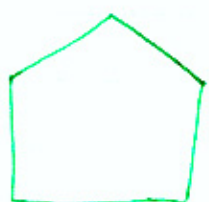
Idea : each polygon  $\rightarrow$   
new "face point"  
new "edge points"



$F$  = Centroid of Polygon

$E$  = Average of center of  $V$ 's and center of  $F$ 's

$$V = \frac{n-2}{n} V_i + \frac{1}{n^2} \sum e + \frac{1}{n^2} \sum f$$



notice, we get  $n$  quads!

Need methods for exact evaluation!

"regular case" = no problem, use B-spline formulas  
gives controls easy to compute "final" position and normals

What about extra-ordinary points?  
subdivide until they become ordinary!

These things are phenomenally popular!

Everything in a PIXAR MOVIE (more or less)

Why?

SUBDIVIDE ANY MESH

any topology

cut, stitch, ... all you like

No cracking

Nice, smooth surfaces

Make creases by modifying subdivision rules

don't move on first few iterations

Add extra-details only where needed

add points only at certain levels!

Evaluate exactly, anywhere!

Joe Storn paper '98