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OCTOBER 16

- project demo signups
- Spike TALK / ART show
- new schedule - no P3 this week

MORE LINEAR TRANSFORMS

LAST TIME

- Coordinate Systems
- Transformations
- Linear Transforms (Samples)
- Composition by multiplication
- Importance of Order
- Post-Multiply notation

Today ≡

Translation // Homogeneous Coordinates

Standard Tricks

Matrices as Transformations / Co-Ordinate Systems

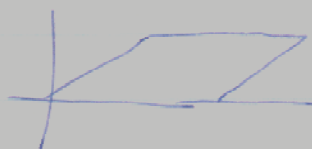
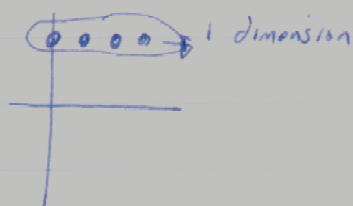
Open GL

3D Transformations

Begin: Linear Transforms are Cool -
but can't translate

BIG IDEA: HOMOGENEOUS CO-ORDINATES

TRANSLATION IN N -dimensions = Shear in $N+1$



Idea - take your points, move them away from origin
 distance from origin matters - for now just use unit values

Homogeneous Co-Ords are useful for lots of things!

$$x, y \Rightarrow \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\text{Translation} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

we were careful to preserve this

$$\text{Scale} = \begin{bmatrix} s & & \\ & s & \\ & & 1 \end{bmatrix}$$

$$\text{ROTATE} = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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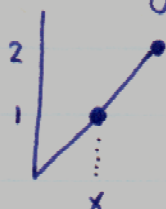
More on this extra-co-ordinate

① call it W

② generally $w=1$

③ mapping from $3D \rightarrow 2D$

$$x, y, w \Rightarrow \frac{x}{w}, \frac{y}{w}$$



bunch of equivalent points
bad things if $w=0$

scale could be

$$\begin{bmatrix} s & & \\ & s & \\ & & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1/s \end{bmatrix}$$

Homogeneous equivalence

$$x, y, w \Rightarrow \frac{x}{w}, \frac{y}{w}, 1$$

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TRANSFORMATION MATRICES AS CO-ORD SYSTEMS

- ① where does origin go $\begin{matrix} 0 & 0 & 1 \end{matrix}$
 ② where does axis go $\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$

Do we change the co-ord system, or the object?
 It's the same!

World T S R Q X



Composition: Multiply Matrices

Standard tricks-

- ~~move~~ rotate around arbitray point
- move center to origin
- rotate around origin
- move center back

$T_c R T_{-c} X$

Scale along an axis

- move point to origin
- align axis w/ co-ordinate axis
- scale
- rotate back
- translate back

Inverses

EASY

GENERAL

(8)

(5)

What does this do in software?

OpenGL

Stated drawing system

current color / style

current co-ordinate system

↳ matrix from co-ord system (object) $\Rightarrow$ screen
(simplification)

stack

multiply, load, clear