Lecture 11 - October 9, 2001

Raster Algorithms

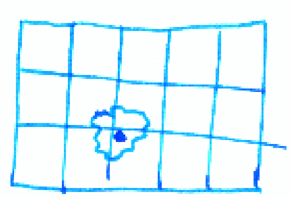
We want to think about geometry
we need to draw pixels

Drawing a Point -
- What is a point -
  - location
  - no extent <- how can you see it?

Draw a region centered around a location

Co-ordinate Systems - wait till next time

How does this relate to our sampled world -



fine if centered and same size
if not?
- pick closest pixel
- point sample pixel centers] Aliasing
- area coverage
- smart sampling] partial values, trade brightness for resolution, blurry

What do we do?

Be aware of these issues

If details matter, design so things line up on pixel boundaries (font hinting)

Use these half-way representations if we really care

smooth and blurry vs. sharp, jaggy, ...

Points don't matter much

LINE DRAWING

IS (WAS IMPORTANT)

How we draw on paper or Vector Scope

if we could use cheap Raster Devices to

replace expensive Vector ones.......

The problem =

given x_1, y_1 x_2, y_2 (integer-pixel locations)

draw set of pixels that represent this line

How to do?

Straw MAN $y = mx + b$

FOR $X = X_1$ to X_2

$Y = MX + B$

SET (X, Y)

PROBLEMS:

GAPS (slope > 1) $\Rightarrow$ loop over choice

LOTS OF MATH

DDA

if $m \leq 1$ (and $m \geq 0$)

$\Delta y = m$, $y = y_1$ ← needs floating point

for $x = x_1$ to x_2

plot x, y ← integer rounding

$y \mathrel{+}= \Delta y$

Brezenham -

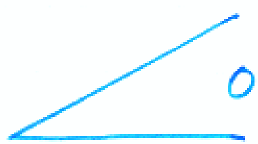
draw lines with no floating point

Midpoint - Method ←

cleaner derivation, similar algorithm

extends to circles

1. deal with one octant - get rest by symmetry



$0 \leq$ slope ≤ 1

2. loop over x pixels (one point per column)

3. if x, y then either $x+1, y$ or $x+1, y+1$

trick - need to pick (need some criterion)

4.
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  Y = Y1
  FOR X = X1 to X2
      Plot x,y
      IF TEST Y = Y+1
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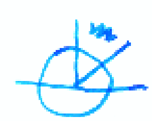
5. TRICK - NEED TEST

WORKS FOR CIRCLES AS WELL - Different TEST

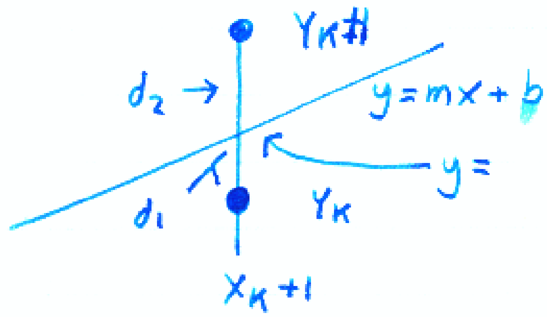


MID-POINT TEST

$Y = MX + B$

SAY WE JUST PLOTTED X_K, Y_K

Which is closer to the line X_K+1, Y_K or X_K+1, Y_K+1



$d_1 = y - y_K$

$d_2 = y_K + 1 - y$

if $d_1 - d_2 < 0$ pick y_{K+1}

ΔD

$$\Delta D = \overbrace{y - y_K}^{d_1} - \overbrace{(y_K + 1 - y)}^{d_2}$$

$$y = m(X_K + 1) + B$$

$$2\left(\underset{\uparrow \frac{\Delta Y}{\Delta X}}{m}(X_K + 1) + b\right) - 2y_K - 1$$

$$\Delta d = 2\frac{\Delta Y}{\Delta X}(X_K + 1) + 2b - 2y_K - 1$$

SINCE $\Delta X > 0$, $\Delta X \Delta d$ has same sign all we care about

$$\Delta D \Delta X = 2\Delta Y X_K - 2\Delta X Y_K + C$$

$$C = 2\Delta Y + \Delta X(2B - 1)$$

CALL THIS P (the decision variable)

SAY WE KNOW P_K

$$P_{K+1} = P_K + 2\Delta Y - 2\Delta X\,(Y_{K+1} - Y_K)$$

either 1 or 0 based on P_K

So ....

$P_K = 2 * \Delta Y + \Delta X$

$Y = Y_1$

FOR ~~X=~~ X_1 to X_2

  SET X, Y

  If $P_K > 0$

    $Y = Y + 1$

    $P_K += 2\Delta Y - 2\Delta X$

  ELSE

    $P_K += 2\Delta Y$

COOL!

No Division

No Floating POINT

No GAPS

EXTENDS TO CIRCLES

UNCOOL :

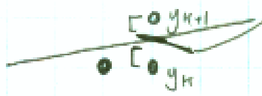
- ALIASING
  - JAGGIES
  - LINES Thin as they approach 45°
  - Equal area / unequal area - thick primitive

FILLED TRIANGLES

- SCAN CONVERSION
- HORIZONTAL ROWS
  - Brezenham's (midpoint) to do begin / end
- DETAILS UNIMPORTANT
  - Hardware implements
  - Very common and important

Filled regions

- Break into triangles
- Polygon scan conversion
  - edge lists
- Flood



$$y = mx + b$$

$$d_2 = y_k + 1 - y$$ if $d_1 - d_2 < 0$, pick y_{k+1}

$$d_1 = y - y_k$$

$$y = m(x_k + 1) + b$$

$$d_1 - d_2 = y - y_k - ((y_k + 1) - y) = 2y - 2y_k - 1$$

$$= 2(m(x_k + 1) + b) - 2y_k - 1$$

$$2\frac{\Delta Y}{\Delta X}(x_k + 1) + 2b - 2y_k - 1$$

V

decision variable

multiply by ΔX (since its positive)

$$= 2\Delta Y(x_k + 1) + 2b - 2y_k - 1$$

new decision var
$p = V\Delta X$

$$= 2\Delta Y\, X_k - 2\Delta X\, Y_k + C$$

C ← $2\Delta y + \Delta X(2b - 1)$

so, if we know p_k,

$$p_{k+1} = 2\Delta Y - 2\Delta X(Y_{k+1} - y) + p_k$$

($Y_{k+1} - y$) ← either 1 or 0

Start $\equiv$ $x_k = x_1$, $y_k = y_1$

$$p @ 0 = 0$$

$$p @ k = 2\Delta y - \Delta x$$

Step $x_k \rightarrow x_2$

next point is $x_k + 1, y_k$ if $p_k < 0$

$$p_k + 1 = p_k + 2\Delta y$$

else $x_k + 1, y_k + 1$, $p_{k+1} = p_k + 2\Delta y - 2\Delta X$

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LD 1

Line Drawing

$y = mx + b$

straw man -

for $x = x_1$ to x_2

$y = mx+b$

plot x,y

better DDA

if $m < 1$

$\Delta y = m$

$y = y_1$

for $x = x_1$ to x_2

plot x,y

$y \mathrel{+}= \Delta y$

if $m > 1$

$\Delta x = \frac{1}{m}$

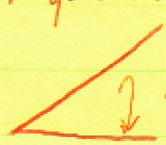
no multiplies

requires floats

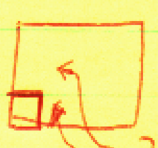
Bresenham's Line Algorithm

only integer math

for 1 quadrant (get others by symmetry)



45° (slope positive < 1)



2 choices

if know x_k, y_k then x_k+1, $y = y_k$ or y_k+1

must decide which is closer x_k+1, y_k, x_k+1, y_k+1

which is closer to $m(x_k+1)+b$?

let d_1 = distance y_k to line

d_2 = distance y_{k+1} to line

if $d_1 - d_2$ if positive, pick y_k, negative pick y_{k+1}

$$d_1 = \left| y_k - \left(\overset{y}{m(x_k+1)+b} \right) \right| \qquad \text{in quadrant, } d_1 < y < d_2$$

$$d_2 = \left| y_k+1 - \left(\overset{y}{m(x_k+1)+b} \right) \right|$$

$$d_1 - d_2 = y - y_k - \left((y_{k+1}) - y \right) = 2y - 2y_k - 1$$

$$2\left(m(x_k+1)+b \right) - 2y_k - 1$$

$m = \Delta y / \Delta x$

$$= 2\frac{\Delta y}{\Delta x}(x_k+1) + 2b - 2y_k - 1$$

$$\Delta x \quad \Delta d = \cancel{2\Delta y x_k + 2\Delta y + 2b - 2\Delta x}$$

↑

is positive, so $\Delta x \, \Delta d$ has same sign as Δd

$$= 2\Delta y \cdot x_k - 2\Delta x \, y_k + c$$

$$c = 2\Delta y + \Delta x(2b-1)$$

if we know p_k, ← either 0 or 1

$$p_{k+1} = p_k + 2\Delta y - 2\Delta x (y_{k+1} - y)$$

LD3

Why is this important
  No division
  No multiplies in inner loop - just adds
  Some Precomputing / 8 different versions (octants)

Generalizes to other shapes
  implicit function
    0 on shape
    + outside
    - inside
  circle (also has slope < 1 in 1st octant)