### CS559 Midterm Exam

October 28, 2009, 7:15pm-9:15 pm

This exam is closed book and closed notes.

You will have the entire period (until 9:15) to complete the exam, however it was designed to take less time.

Please write your name and WISCID on **every page**! (we may unstaple the exams for grading). Please do this first (and check to make sure you have all of the pages)

Write numerical answers in fractional form or use radicals (square root symbols) – we would prefer to see than .866. You should not need a calculator for this exam.

Unless otherwise noted, assume that everything is a right-handed coordinate system and that angles are measured counter clockwise. E.g. to find the direction of rotation, point your thumb along the axis and curl your fingers.

If you need extra space, use the back of a page, but clearly mark what everything is. We may look at your work to determine partial credit. Not providing an answer is not the same as giving a wrong answer. For “forced choice” questions (multiple choice, true false, etc.), wrong answers will be penalized more than no answers.

If you feel a question is ambiguous or requires more information for you to make a good answer, you can state any additional assumptions you needed to arrive at your answer. However, you should probably avoid this: usually it is more a sign of a lack of thought about the question or understanding of the material.

Total : X points

### Question 1: Lighting (10 parts, +1 for each correct, -1 for each wrong)

While we didn’t mention it in class, you should know from the readings that a “directional light source” is one where all rays from the source are parallel (in the same direction). This is in contrast to a point light source, where all light rays come from a single point location.

Consider a sphere lit from above with a directional light source (so the light is pointing downwards). Consider using the Specular/Diffuse/Ambient lighting model (sometimes called the Phong model) as used by OpenGL or discussed in class and the readings.

You should also know that for a sphere, the normals always point outward (the normal at a point is the vector from the center to the point).

True or False (for each one):

1. \_\_\_\_\_ If the sphere is a purely **diffuse** material, it will appear the same brightness everywhere.
2. \_\_\_\_\_ If the sphere is a purely **diffuse** material, there will be a single point that will be the brightest.
3. \_\_\_\_\_ For any specific point on the top half of the sphere, if the sphere is a purely **diffuse** material, that point will appear the same brightness no matter what direction you look from.
4. \_\_\_\_\_ For any specific point on the top half of the sphere, if the sphere is a purely **specular** material, that point will appear the same brightness no matter what direction you look from.
5. \_\_\_\_\_ If the sphere is a purely **specular** material, the brightest point depends on where you look from.
6. \_\_\_\_\_ If the sphere is a purely **diffuse** material, the brightest point depends on where you look from.
7. \_\_\_\_\_ If you raise the **ambient** term, the position of the brightest spot on the sphere may change (i.e. a different point on the sphere may be the brightest).

The following questions are about the lighting model, not just for the sphere:

1. \_\_\_\_\_ This lighting model includes shadows.
2. \_\_\_\_\_ This lighting model includes reflections of other objections.
3. \_\_\_\_\_ This lighting model treats each light source separately, but we can combine them by adding their contributions separately.

### Question 2: Interpolating Cubics (8 pts)

Sketch a Catmull-Rom (e.g. Cardinal Cubic with tension 0) spline through the following sequence of points (the points are in order as numbered 1-7, ignore points 8 and 9):



### Question 3: Bezier & Cardinal Curves (12 pts – 4 for each part)

A: A Bezier Quadratic curve segment has its control points at (0,0), (12,0), (12,24). Give the control points for a Bezier Cubic curve segment that covers the same curve.

B: What is the position of this cubic curve at parameter value u=.25?

C: If you were to create a Catmull-Rom cubic segment that is identical to this curve, where would the 4 control points be?

### Question 4 Bezier Curve properties (10 parts, 1pt each):

A curve is made from 2 connected cubic Bezier segments (call them A and B).

We’ll name the control points A1, A2, A3, A4, B1, B2, B3, B4 (in order).

The curves are connected so that the end of the first segment connects to the end of the second segment. (that is A4=B1).

Other than the 2 points that are the same, the points are strictly increasing in the value of their X coordinate (so A1.x < A2.x < A3.x < A4.x = B1.x < B2.x < B3.x < B4.x).

The Control points are also arranged such that A3, A4, B1 and B2 are all on a line.

While the curves are in 3D, the Z values of all of the points are zero.

For each of the following statements determine if they are:

ALWAYS true (for any positioning of the points subject to the above)

NEVER true (for any positioning of the points subject to the above)

SOMETIMES true and sometimes false (depending on the positions of the points)

(write A, N or S)

1. \_\_\_\_\_ The curve is C(0) continuous
2. \_\_\_\_\_ The curve is G(1) continuous
3. \_\_\_\_\_ The curve is C(1) continuous
4. \_\_\_\_\_ The curve is C(2) continuous
5. \_\_\_\_\_ If we transform the control points to where they appear on the screen and draw a 2D Bezier curve through those points, it would be the equivalent to us drawing the curve in 3D and figuring out every point gets projected to on the screen, if we were using a standard camera transform and **perspective** projection.
6. \_\_\_\_\_ If we transform the control points to where they appear on the screen and draw a 2D Bezier curve through those points, it would be the equivalent to us drawing the curve in 3D and figuring out every point gets projected to on the screen, if we were using a standard camera transform and **orthographic** projection.
7. \_\_\_\_\_ The curve lies entirely in the plane Z=0.
8. \_\_\_\_\_ The curve crosses the Y=0 plane 5 or less times.
9. \_\_\_\_\_ The curve crosses the Y=0 plane exactly 4 times.
10. \_\_\_\_\_ The curve crosses the X=0 plane more than once.

### Question 5: Transformations (10 pts)

Imagine a simple OpenGL-like graphics toolkit. All of the transformation operations are like OpenGL in that there is a matrix stack and the command affect the top of it. Rotation is measured in degrees counterclockwise. For each of the little programs below, assume that the programs start out with the identity matrix on the stack.

The draw square command draws a unit square (0,1) in the 1st quadrant.

Here is an example program and its resulting picture:

|  |  |
| --- | --- |
| Draw SquarePushMatrixScale(2,2)Translate(2,1)Scale(-1,1)Draw SquarePopMatrixRotate(90)Translate(-3,0)Draw Square | simple1 |

**A:** If we forgot the “Push Matrix” and “PopMatrix” commands in the example program, the last square would appear in a different place. Draw it on the example above.

**B:** If we had forgotten the “PushMatrix” and “Pop Matrix” commands (as in part A), and wanted to “reset” the transformation back to the identity at the end, we could add the following 3 lines of code. (in other words, if we added a DrawSquare command, it would place the square where the first line of the program does

Specify what the values of A, SX, SY, TX, TY should be:

 Rotate(A)

 Scale (SX, SY)

Translate (TX, TY)

### Question 6: Rotations (10 pts, 4+2+ 4\*1)

1. A 2D linear transformation is a **rotation** that transforms the point (0,5) to (3, 4). Give the 2x2 matrix for this rotation.
2. How many right answers are there to part A?

Matrix Sudoku – In each case, you are given a 3x3 matrix with only some of the values filled in. You know that it’s a 3x3 rotation matrix.

True or False:

1. If you know all the numbers in one **row**, and 2 of the numbers on any other **row**, you can figure out the other 4 numbers.
2. If you know one **column** of a 3x3 matrix, and you know 1 of the other numbers is 1, you can figure out the rest.
3. If you know the 3 numbers above or below the diagonal, you can figure out the rest.
4. If you know any 3 numbers off the diagonal, you can figure out the rest.

### Question 7 (10pts)

Assume that the x-axis (for the world) points east, the y-axis points north, and the z-axis points up. (this is a right-handed coordinate system)

Imagine a pyramid with a square base placed with its corner at the origin. Each of the triangular sides of the pyramid is painted a different color. The length of a side of the pyramid is 2 units, so the top of the pyramid is at (1,1,1).



Assume that the film plane or screen is the “normal” xy plane with x going to the right and y going up. The center of the film plane is at the origin, and the viewer sights down the negative Z axis.

5A: Give an orthographic projection matrix (a 4x4 matrix) that gives a view of the white side of the pyramid. Don’t worry about depth in the viewing volume other than that objects that are farther away from the viewer should be further along the viewing axis.



5B: Give a Lookfrom/Lookat/VUP (the positions of 2 points and one vector, all in 3D) that specifies a camera that would draw a red triangle pointing to the right, with its tip at the origin.



### Question 8 (10 pts)

Describe in words what each of these 4x4 homogeneous transformation matrices does to objects (when they get brought back into 3D):

1. $\begin{matrix}1&0&0&3\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{matrix}$
2. $\begin{matrix}3&0&0&0\\0&3&0&0\\0&0&3&0\\0&0&0&3\end{matrix}$
3. $\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&3\end{matrix}$
4. $\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&1&1\end{matrix}$
5. $\begin{matrix}1&0&1&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{matrix}$

### Question 9: (4 Points)

For each of the properties, mark P if it is true of a perspective projection, O if it is true of an orthographic projection, B if it is true of both orthographic and projective, and N if it is true of neither. (use P or O only if the property is only true of one type of transformation).

\_\_\_ Straight lines are mapped to straight lines

\_\_\_ An object can be viewed from its front, side or top

\_\_\_ Far away objects appear the same size as closer ones

\_\_\_ Requires homogeneous coordinates in order for it to be encoded into a linear transformation.

**Question 10: Composition (2 Points)**

Given a rotation matrix **R** and a translation matrix **T**, the composition **R**-1**TR** is equivalent to:

1. A rotation
2. A translation
3. A shear
4. A scale
5. None of the above

Question 11: Z-buffer (6 points)

Besides performance (which is a tricky thing, since so many different factors influence it) list two situations where the order that triangles are drawn matter when using a Z-Buffer.

1.

2.

### Question 12: Arc-length parameterizations and blending functions (8pts)

Here is a curve.
It is piecewise linear, and interpolates points p1,p2,p3,p4 and p5.
It is arclength parameterized.
Its parameter range is from 0 to 5.



What is the parameter value when the curve is at (2,1)?

When the parameter has value 1.5, what is the curve’s position?

Sketch the blending function for P3:

