CS 559: Computer Graphics

Homework 3: Solutions

Question 1:

The 2D edge-detect (high-pass) filter that we looked at in class has the following form if you ignore the constant:

-1	-2	-1
-2	12	-2
-1	-2	-1

a. What is the output if you filter the image below? Only apply the filter at places where all the underlying pixels exist, so you end up with a 4×1 result. *[2 points]*

1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0

0 4 -4 0

b. What is the output if you filter the image below (giving a 6×1 output image)? *[2 points]*

1	1	1	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	1	0	0	0	0	0

0 1 5 -5 -1 0

c. Does the filter respond more to a diagonal or vertical edge? *[1 point]*

The filter responds more to the diagonal edge, with the response being both higher and wider.

d. An alternate edge-detect filter is given below. What is its output on each of the above images (you should have a 4×1 and a 6×1 answer)? *[2 points]*

-1	2	-1
-1	2	-1
-1	2	-1

0 3 -3 0

0 1 0 0 -1 0

e. Does this filter respond more to vertical or diagonal edges? *[1 point]*

This filter responds more to vertical edges.

f. Design a 3×3 filter that responds to diagonal edges like that of part (b), but gives no response to a vertical edge. *[3 points]*

A filter with a strong diagonal component will do the job:

$$\begin{matrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & -1 & -1 \end{matrix}$$

Note that this filter does not respond to all diagonal edges, only those running bottom-left to top-right. Filters like these are used in computer vision to find edges at particular orientations, and there are also neural equivalents in our retina as part of early human vision processing.

Question 2:

You wish to use compositing operations to perform a stencil operation. You have a foreground image, f, that you wish to place into a background image, b, only at places where a stencil mask, s, has a particular α value. For example, of the foreground image is all white with $\alpha = 1$, the background is all black with $\alpha = 1$ and the stencil has holes for a word, inserting the foreground into the background would result in a white word on a black background.

a. Which α value would you use for the parts of the stencil that represent holes? Which value would you use for the rest? (There are two good answers to this question.) *[1 point]*

   You could use $\alpha = 0$ for the holes and $\alpha = 1$ for the solid parts. Call this solution A. Alternatively you could use $\alpha = 1$ for the holes and $\alpha = 0$ for the solid parts. Call this solution B.

b. You plan to use two compositing operations to combine the images, with the form $(f\ \mathbf{op}_1\ s)\ \mathbf{op}_2\ b$, where brackets indicate precedence. Which compositing operations would you use for $\mathbf{op}_1$ and $\mathbf{op}_2$? *[4 points]*

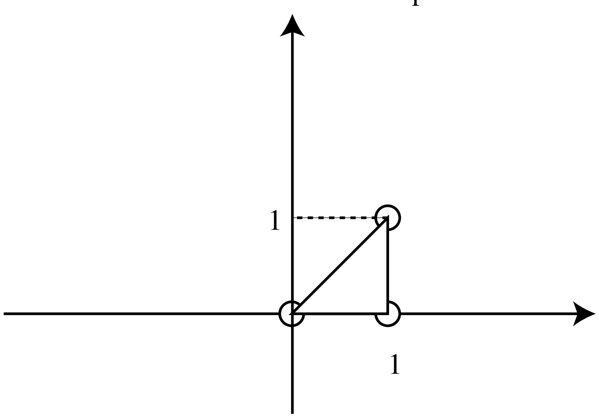
   *If you chose solution A, then you want $\mathbf{op}_1$ to be **out** and $\mathbf{op}_2$ to be **over**. If you chose solution B, then $\mathbf{op}_1$ should be **in** and $\mathbf{op}_2$ should be **over**. The first operation selects the parts of f that will go through the stencil, and the second operation inserts them into b.*

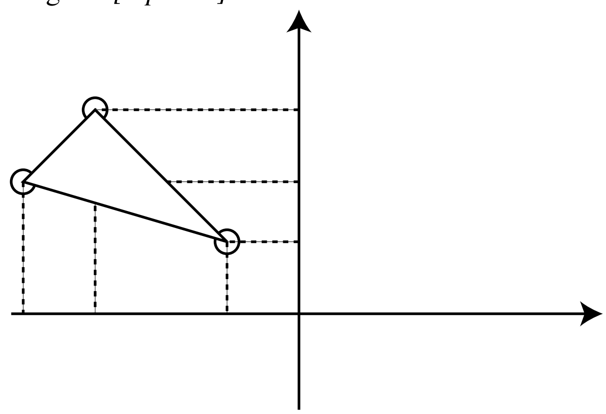
Question 3:

It takes three points to define an affine transformation in 2D. Say that the point $(1,1)$ goes to $(-\frac{4}{\sqrt{2}}, \frac{2}{\sqrt{2}})$, that $(1,0)$ goes to $(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})$, and that the point $(0,0)$ goes to $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. Assume that the affine transformation is described by the following homogeneous matrix equation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{xx} & a_{xy} & b_x \\ a_{yx} & a_{yy} & b_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

a. Draw one sketch showing the locations of the points before the transformation, and one showing their locations after. Join the points to form a triangle in each figure. *[2 points]*





b. Write out six linear equations involving the unknowns in the matrix equation above and the coordinates of the given points. *[2 points]*

$$\begin{array}{ccccccc} a_{xx} & + & a_{xy} & + & b_x & = & -\frac{4}{\sqrt{2}} \\ a_{yx} & + & a_{yy} & + & b_y & = & \frac{2}{\sqrt{2}} \\ a_{xx} & & & + & b_x & = & -\frac{3}{\sqrt{2}} \\ a_{yx} & & & + & b_y & = & \frac{3}{\sqrt{2}} \\ & & & & b_x & = & -\frac{1}{\sqrt{2}} \\ & & & & b_y & = & \frac{1}{\sqrt{2}} \end{array}$$

c. Solve the equations to find the unknowns and hence write out the transformation matrix. *[3 points]*

$$\begin{bmatrix} -\frac{2}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 \end{bmatrix}$$

d. Determine from your sketch a sequence of rotations, scalings or translations required for the transformation. *[2 points]*

There are multiple answers, but one of the simplest is: Scale by 2 in the x direction, translate by 1 in the x direction, rotate clockwise by 135 degrees.

e. Write out the transformation matrices for your sequence in part (d) and compose them to verify your answer to part (c). *[4 points]*

$$\begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & 1 \end{bmatrix}$$

Question 4:

You wish to construct a view that has the origin of the image plane at $(4, 0, 4)$, has the point $(0, 0, 2)$ appearing in the center of the image, and has the direction $(0, 0, 1)$ appearing to be up in the image. The following questions use the notation from lecture 10 for constructing the world to view matrix.

a. What is the vector c? *[1 point]*

$(4, 0, 4)$

b. What is the vector n? *[1 point]*

$\|(4, 0, 4) - (0, 0, 2)\| = (\frac{2}{\sqrt{5}}, 0, \frac{1}{\sqrt{5}})$

c. What is the vector u? *[1 point]*

$(0, 0, 1) \times n = (0, \frac{2}{\sqrt{5}}, 0)$ *After normalizing,* $u = (0, 1, 0)$

d. What is the vector v? *[1 point]*

$v = n \times u = (-\frac{1}{\sqrt{5}}, 0, \frac{2}{\sqrt{5}})$

e. What is the 4×4 world to view matrix? *[2 points]*

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{1}{\sqrt{5}} & 0 & \frac{2}{\sqrt{5}} & -\frac{4}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & 0 & \frac{1}{\sqrt{5}} & -\frac{12}{\sqrt{5}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$